Robust Control Toolbox™

User's Guide

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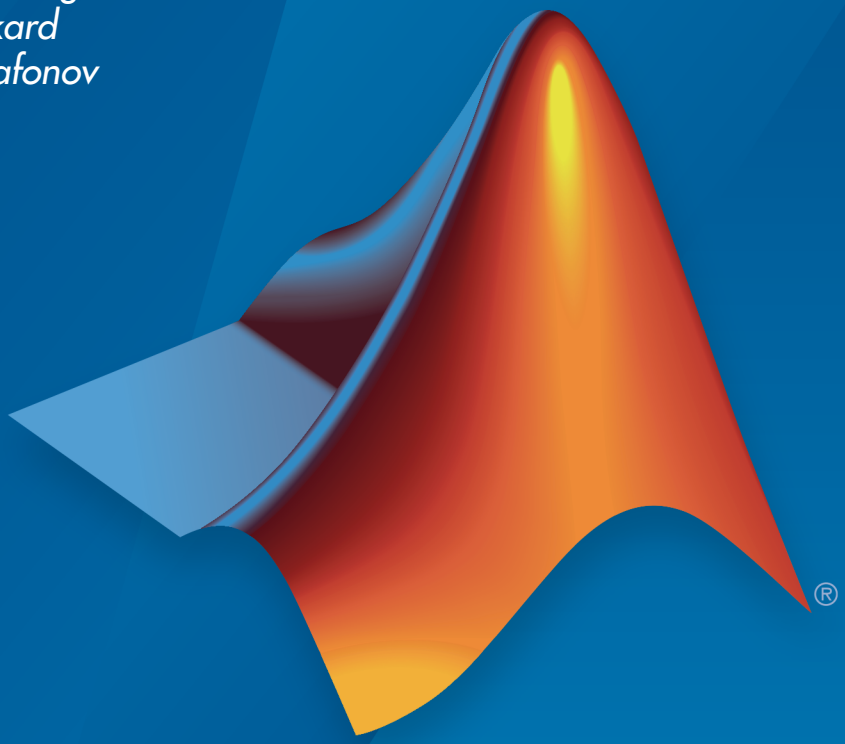

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Building Uncertain Models

Introduction to Uncertain Elements

Uncertain elements (also called uncertain Control Design Blocks) are the building blocks used to form uncertain matrix objects and uncertain system objects. There are 5 types of uncertain elements:

Function	Description
`ureal`	Uncertain real parameter
`ultidyn`	Uncertain, linear, time-invariant dynamics
`ucomplex`	Uncertain complex parameter
`ucomplexm`	Uncertain complex matrix
`udyn`	Uncertain dynamic system

All of the elements have properties, which are accessed through `get` and `set` methods. This `get` and `set` interface mimics the Control System Toolbox™ and MATLAB® Handle Graphics® behavior. For instance, `get(a,'PropertyName')` is the same as `a.PropertyName`, and `set(b,'PropertyName',Value)` is the same as `b.PropertyName = value`. Functionality also includes tab-completion and case-insensitive, partial name property matching.

For `ureal`, `ucomplex` and `ucomplexm` elements, the syntax is

```
p1 = ureal(name, NominalValue, Prop1, val1, Prop2, val2,...);
p2 = ucomplex(name, NominalValue, Prop1, val1, Prop2, val2,...);
p3 = ucomplexm(name, NominalValue, Prop1, val1, Prop2, val2,...);
```

For `ultidyn` and `udyn`, the `NominalValue` is fixed, so the syntax is

```
p4 = ultidyn(name, ioSize,  Prop1, val1, Prop2, val2,...);
p5 = udyn(name, ioSize,  Prop1, val1, Prop2, val2,...);
```

For `ureal`, `ultidyn`, `ucomplex` and `ucomplexm` elements, the command `usample` will generate a random instance (i.e., not uncertain) of the element, within its modeled range. For example,

```
usample(p1)
```

creates a random instance of the uncertain real parameter `p1`. With an integer argument, whole arrays of instances can be created. For instance

```
usample(p4,100)
```

generates an array of 100 instances of the `ultidyn` object `p4`. See “Generate Samples of Uncertain Systems” on page 1-50 to learn more about `usample`.

Related Examples

- “Uncertain Real Parameters” on page 1-4
- “Uncertain Matrices” on page 1-23
- “Uncertain LTI Dynamics Elements” on page 1-12

Uncertain Real Parameters

An uncertain real parameter, `ureal`, is the Control Design Block that represents a real number whose value is uncertain.

Properties of Uncertain Real Parameters

Uncertain real parameters have a name (the `Name` property), and a nominal value (the `NominalValue` property). Several other properties (`PlusMinus`, `Range`, `Percentage`) describe the uncertainty in parameter values.

All properties of a `ureal` can be accessed through `get` and `set`. The properties are:

Properties	Meaning	Class
`Name`	Internal name	`char`
`NominalValue`	Nominal value of element	`double`
`Mode`	Signifies which description (from `'PlusMinus'`, `'Range'`, `'Percentage'`) of uncertainty is invariant when `NominalValue` is changed	`char`
`PlusMinus`	Additive variation	`scalar or 1x2 double`
`Range`	Numerical range	`1x2 double`
`Percentage`	Additive variation (% of absolute value of nominal)	`scalar or 1x2 double`
`AutoSimplify`	`'off' | {'basic'} |'full'`	`char`

The properties `Range`, `Percentage` and `PlusMinus` are all automatically synchronized. If the nominal value is 0, then the `Mode` cannot be `Percentage`. The `Mode` property controls what aspect of the uncertainty remains unchanged when `NominalValue` is changed. Assigning to any of `Range`/`Percentage`/`PlusMinus` changes the value, *but does not* change the mode.

The `AutoSimplify` property controls how expressions involving the real parameter are simplified. Its default value is `'basic'`, which means elementary methods of simplification are applied as operations are completed. Other values for `AutoSimplify` are `'off'` (no simplification performed) and `'full'` (model-reduction-like techniques

are applied). See “Simplifying Representation of Uncertain Objects” on page 1-46 to learn more about the `AutoSimplify` property and the command `simplify`.

If no property/value pairs are specified, default values are used. The default `Mode` is `PlusMinus`, and the default value of `PlusMinus` is `[-1 1]`. Some examples are shown below. In many cases, the full property name is not specified, taking advantage of the case-insensitive, partial name property matching.

Create Uncertain Real Parameters

This example shows how to create uncertain real parameters, modify properties such as range of uncertainty, and sample uncertain parameters.

Create an uncertain real parameter, nominal value 3, with default values for all unspecified properties (including plus/minus variability of 1).

```
a = ureal('a',3)
```

```
a =

  Uncertain real parameter "a" with nominal value 3 and variability [-1,1].
```

View the properties and their values, and note that the `Range` and `Percentage` descriptions of variability are automatically maintained.

```
get(a)
```

```
            Name: 'a'
    NominalValue: 3
            Mode: 'PlusMinus'
           Range: [2 4]
       PlusMinus: [-1 1]
      Percentage: [-33.3333 33.3333]
    AutoSimplify: 'basic'
```

Create an uncertain real parameter, nominal value 2, with 20% variability. Again, view the properties, and note that the `Range` and `PlusMinus` descriptions of variability are automatically maintained.

```
b = ureal('b',2,'Percentage',20)
```

```
get(b)
```

```
b =

  Uncertain real parameter "b" with nominal value 2 and variability [-20,20]%.

            Name: 'b'
    NominalValue: 2
            Mode: 'Percentage'
           Range: [1.6000 2.4000]
       PlusMinus: [-0.4000 0.4000]
      Percentage: [-20 20]
    AutoSimplify: 'basic'
```

Change the range of the parameter. All descriptions of variability are automatically updated, while the nominal value remains fixed. Although the change in variability was accomplished by specifying the `Range`, the `Mode` is unaffected, and remains `Percentage`.

```
b.Range = [1.9 2.3];
get(b)
```

```
            Name: 'b'
    NominalValue: 2
            Mode: 'Percentage'
           Range: [1.9000 2.3000]
       PlusMinus: [-0.1000 0.3000]
      Percentage: [-5.0000 15.0000]
    AutoSimplify: 'basic'
```

As mentioned, the `Mode` property signifies what aspect of the uncertainty remains unchanged when `NominalValue` is modified. Hence, if a real parameter is in `Percentage` mode, then the `Range` and `PlusMinus` properties are determined from the `Percentage` property and `NominalValue`. Changing `NominalValue` preserves the `Percentage` property, and automatically updates the `Range` and `PlusMinus` properties.

```
b.NominalValue = 2.2;
get(b)
```

```
            Name: 'b'
    NominalValue: 2.2000
            Mode: 'Percentage'
```

```
          Range: [2.0900 2.5300]
       PlusMinus: [-0.1100 0.3300]
     Percentage: [-5.0000 15.0000]
   AutoSimplify: 'basic'
```

Create an uncertain parameter with an asymmetric variation about its nominal value. Examine the properties to confirm the asymmetric range.

```
c = ureal('c',-5,'Percentage',[-20 30]);
get(c)
```

```
            Name: 'c'
    NominalValue: -5
            Mode: 'Percentage'
           Range: [-6 -3.5000]
       PlusMinus: [-1 1.5000]
     Percentage: [-20 30]
   AutoSimplify: 'basic'
```

Create an uncertain parameter, specifying variability with `Percentage`, but force the `Mode` to be `Range`.

```
d = ureal('d',-1,'Mode','Range','Percentage',[-40 60]);
get(d)
```

```
            Name: 'd'
    NominalValue: -1
            Mode: 'Range'
           Range: [-1.4000 -0.4000]
       PlusMinus: [-0.4000 0.6000]
     Percentage: [-40 60]
   AutoSimplify: 'basic'
```

Finally, create an uncertain real parameter, and set the `AutoSimplify` property to `'full'`.

```
e = ureal('e',10,'PlusMinus',[-23],'Mode','Percentage','AutoSimplify','Full')
get(e)
```

```
e =
```

```
  Uncertain real parameter "e" with nominal value 10 and variability [-230,230]%.

           Name: 'e'
   NominalValue: 10
           Mode: 'Percentage'
          Range: [-13 33]
      PlusMinus: [-23 23]
     Percentage: [-230 230]
   AutoSimplify: 'full'
```

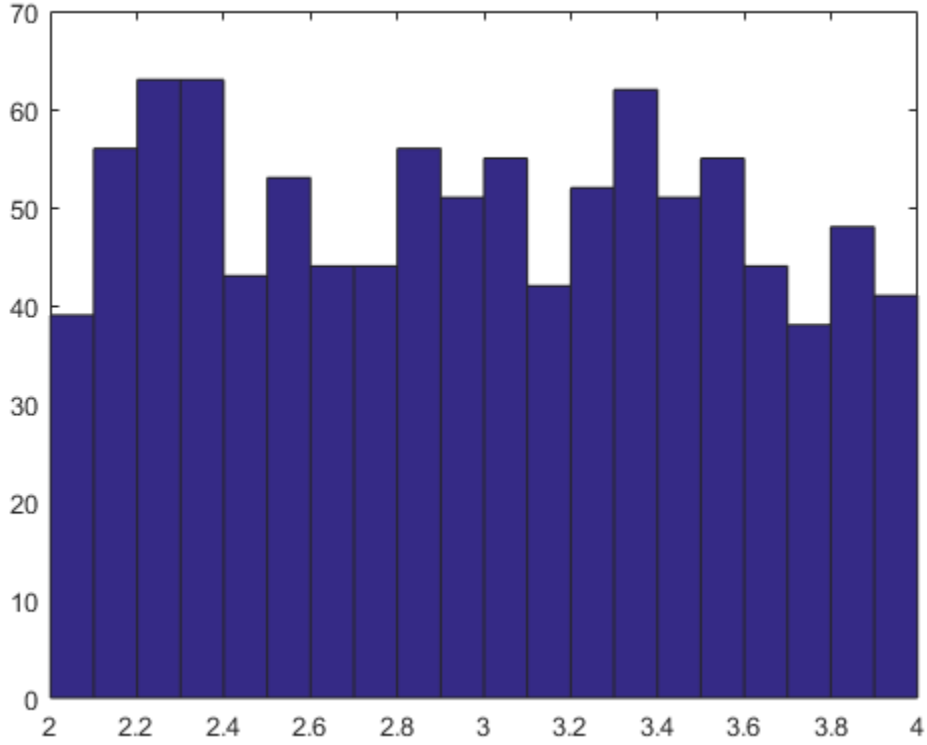
Specifying conflicting values for `Range/Percentage/PlusMinus` when creating a `ureal` element does not result in an error. In this case, the last specified property is used. This last occurrence also determines the `Mode`, unless `Mode` is explicitly specified, in which case that is used, regardless of the property/value pairs ordering.

```
f = ureal('f',3,'PlusMinus',[-2 1],'Percentage',40)
g = ureal('g',2,'PlusMinus',[-2 1],'Mode','Range','Percentage',40)
g.Mode
```

```
f =

  Uncertain real parameter "f" with nominal value 3 and variability [-40,40]%.


g =

  Uncertain real parameter "g" with nominal value 2 and range [1.2,2.8].


ans =

Range
```

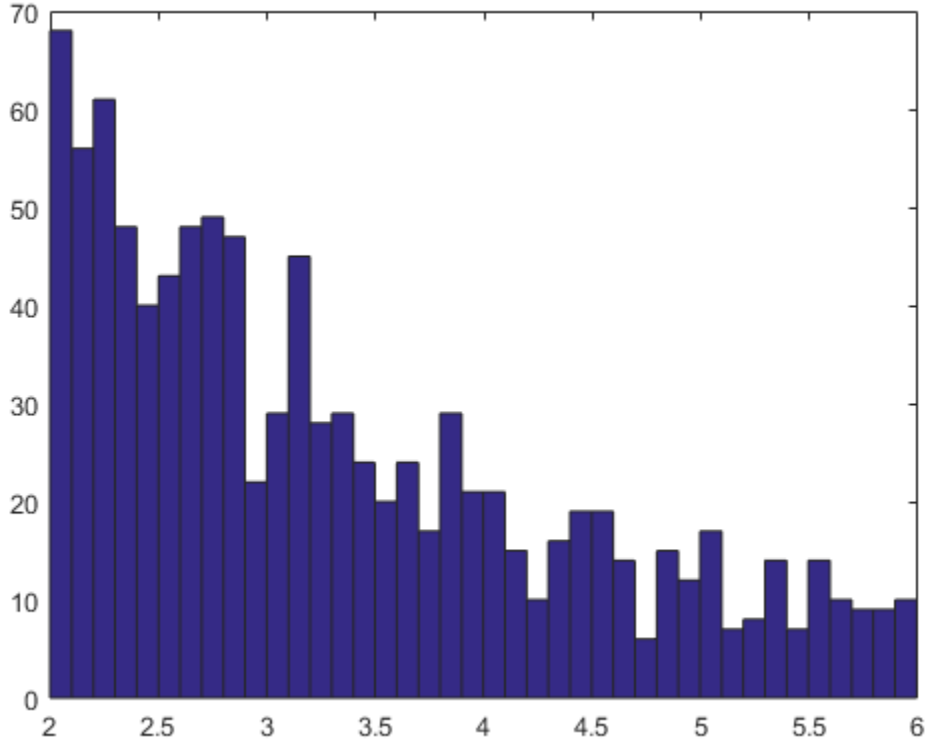
Create an uncertain real parameter, use usample to generate 1000 instances (resulting in a 1-by-1-by-1000 array), reshape the array, and plot a histogram, with 20 bins (within the range of 2 to 4).

```
h = ureal('h',3);
hsample = usample(h,1000);
hist(reshape(hsample,[1000 1]),20);
```



Make the range unsymmetric about the nominal value, and repeat the sampling, and histogram plot (with 40 bins over the range of 2-to-6)

```
h.Range = [2 6];
hsample = usample(h,1000);
hist(reshape(hsample,[1000 1]),40);
```



Note that the distribution is skewed. However, the number of samples less than the nominal value and the number of samples greater than the nominal value are equal (on average). Verify this.

```
length(find(hsample(:) < h.NominalValue))
```

```
ans =

   482
```

```
length(find(hsample(:) > h.NominalValue))
```

```
ans =

   518
```

The distribution used in usample is uniform in the normalized description of the uncertain real parameter. See “Decomposing Uncertain Objects” to learn more about the normalized description.

There is no notion of an empty `ureal` (or any other uncertain element, for that matter). `ureal`, by itself, creates an element named `'UNNAMED'`, with default property values.

See Also

`ureal`

Related Examples

- “System with Uncertain Parameters”
- “Uncertain LTI Dynamics Elements” on page 1-12

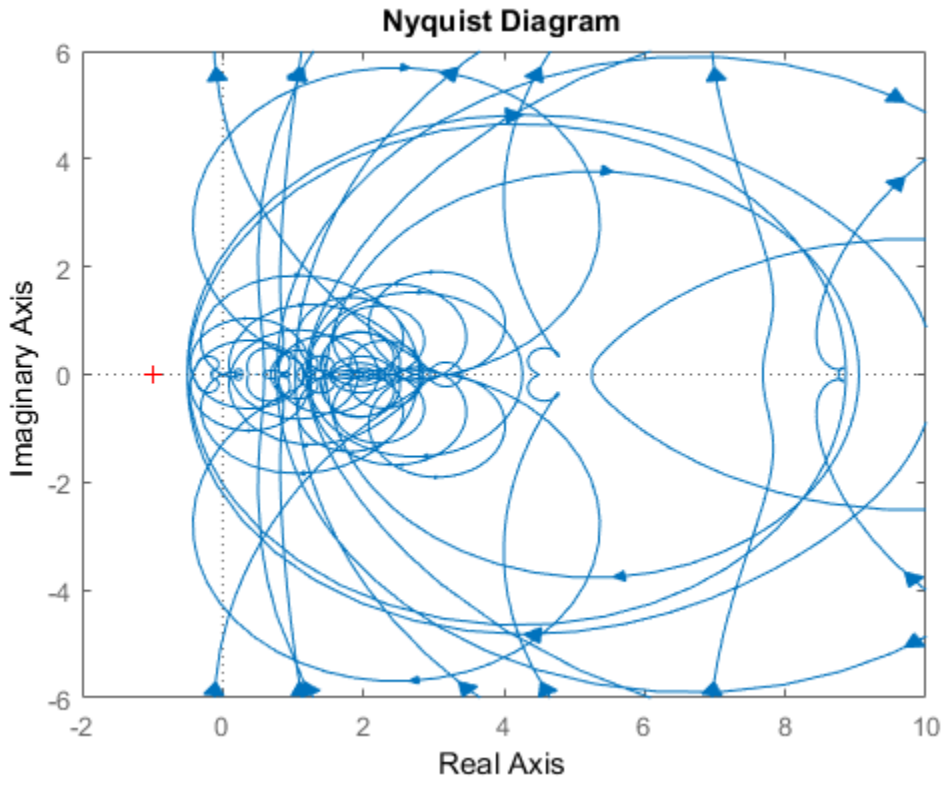
Uncertain LTI Dynamics Elements

Uncertain linear, time-invariant objects, `ultidyn`, are used to represent unknown linear, time-invariant dynamic objects, whose only known attributes are bounds on their frequency response.

Create Uncertain LTI Dynamics

You can create a 1-by-1 (scalar) positive-real uncertain linear dynamics element, whose frequency response always has real part greater than -0.5. Set the `SampleStateDimension` property to 5. Plot a Nyquist plot of 30 instances of the element.

```
g = ultidyn('g',[1 1],'Type','Positivereal','Bound',-0.5);
g.SampleStateDimension = 5;

nyquist(usample(g,30))
xlim([-2 10])
ylim([-6 6]);
```



Properties of ultidyn Elements

Uncertain linear, time-invariant objects have an internal name (the `Name` property), and are created by specifying their size (number of outputs and number of inputs).

The property `Type` specifies whether the known attributes about the frequency response are related to gain or phase. The property `Type` may be `'GainBounded'` or `'PositiveReal'`. The default value is `'GainBounded'`.

The property `Bound` is a single number, which along with `Type`, completely specifies what is known about the uncertain frequency response. Specifically, if Δ is an `ultidyn` element, and if γ denotes the value of the `Bound` property, then the element represents

the set of all stable, linear, time-invariant systems whose frequency response satisfies certain conditions:

If `Type` is `'GainBounded'`, $\bar{\sigma}\left[\Delta(\omega)\right] \le \gamma$ for all frequencies. When `Type` is `'GainBounded'`, the default value for `Bound` (i.e., γ) is 1. The `NominalValue` of Δ is always the 0-matrix.

If `Type` is `'PositiveReal'`, $\Delta(\omega) + \Delta^*(\omega) \ge 2\gamma \cdot$ for all frequencies. When `Type` is `'PositiveReal'`, the default value for `Bound` (i.e., γ) is 0. The `NominalValue` is always $(\gamma + 1 + 2|\gamma|)I$.

All properties of a `ultidyn` are can be accessed with `get` and `set` (although the `NominalValue` is determined from `Type` and `Bound`, and not accessible with `set`). The properties are

Properties	Meaning	Class
`Name`	Internal Name	`char`
`NominalValue`	Nominal value of element	`See above`
`Type`	`'GainBounded' |'PositiveReal'`	`char`
`Bound`	Norm bound or minimum real	`scalar double`
`SampleStateDimen`	State-space dimension of random samples of this uncertain element	`scalar double`
`SampleMaxFrequen`	Maximum natural frequency for random sampling	`scalar double`
`AutoSimplify`	`'off' | {'basic'} |'full'`	`char`

The `SampleStateDim` property specifies the state dimension of random samples of the element when using `usample`. The default value is 1. The `AutoSimplify` property serves the same function as in the uncertain real parameter.

Time Domain of ultidyn Elements

On its own, every `ultidyn` element is interpreted as a continuous-time, system with uncertain behavior, quantified by bounds (gain or real-part) on its frequency response. However, when a `ultidyn` element is an uncertain element of an uncertain state space model (`uss`), then the time-domain characteristic of the element is determined from the

time-domain characteristic of the system. The bounds (gain-bounded or positivity) apply to the frequency-response of the element. This is explained and demonstrated in .

Interpreting Uncertainty in Discrete Time

The interpretation of a `ultidyn` element as a continuous-time or discrete-time system depends on the nature of the uncertain system (`uss`) within which it is an uncertain element.

For example, create a scalar `ultidyn` object. Then, create two 1-input, 1-output uss objects using the `ultidyn` object as their "D" matrix. In one case, create without specifying sample-time, which indicates continuous time. In the second case, force discrete-time, with a sample time of 0.42.

```
delta = ultidyn('delta',[1 1]);
sys1 = uss([],[],[],delta)
USS: 0 States, 1 Output, 1 Input, Continuous System
  delta: 1x1 LTI, max. gain = 1, 1 occurrence
sys2 = uss([],[],[],delta,0.42)
USS: 0 States, 1 Output, 1 Input, Discrete System, Ts = 0.42
  delta: 1x1 LTI, max. gain = 1, 1 occurrence
```

Next, get a random sample of each system. When obtaining random samples using `usample`, the values of the elements used in the sample are returned in the 2nd argument from `usample` as a structure.

```
[sys1s,d1v] = usample(sys1);
[sys2s,d2v] = usample(sys2);
```

Look at `d1v.delta.Ts` and `d2v.delta.Ts`. In the first case, since `sys1` is continuous-time, the system `d1v.delta` is continuous-time. In the second case, since `sys2` is discrete-time, with sample time 0.42, the system `d2v.delta` is discrete-time, with sample time 0.42.

```
d1v.delta.Ts
ans =
     0
d2v.delta.Ts
ans =
    0.4200
```

Finally, in the case of a discrete-time `uss` object, it is not the case that `ultidyn` objects are interpreted as continuous-time uncertainty in feedback with sampled-data systems. This very interesting hybrid theory is beyond the scope of the toolbox.

See Also

`ultidyn`

Related Examples

- “Uncertain Real Parameters” on page 1-4
- “Uncertain Matrices” on page 1-23
- “Uncertain State-Space Models” on page 1-32

Uncertain Complex Parameters and Matrices

Uncertain Complex Parameters

The `ucomplex` element is the Control Design Block that represents an uncertain complex number. The value of an uncertain complex number lies in a disc, centered at `NominalValue`, with radius specified by the `Radius` property of the `ucomplex` element. The size of the disc can also be specified by `Percentage`, which means the radius is derived from the absolute value of the `NominalValue`. The properties of `ucomplex` objects are

Properties	Meaning	Class
`Name`	Internal Name	`char`
`NominalValue`	Nominal value of element	`double`
`Mode`	`'Range' | 'Percentage'`	`char`
`Radius`	Radius of disk	`double`
`Percentage`	Additive variation (percent of `Radius`)	`double`
`AutoSimplify`	`'off' | {'basic'} | 'full'`	`char`

The simplest construction requires only a name and nominal value. Displaying the properties shows that the default `Mode` is `Radius`, and the default radius is 1.

```
a = ucomplex('a',2-j)
```

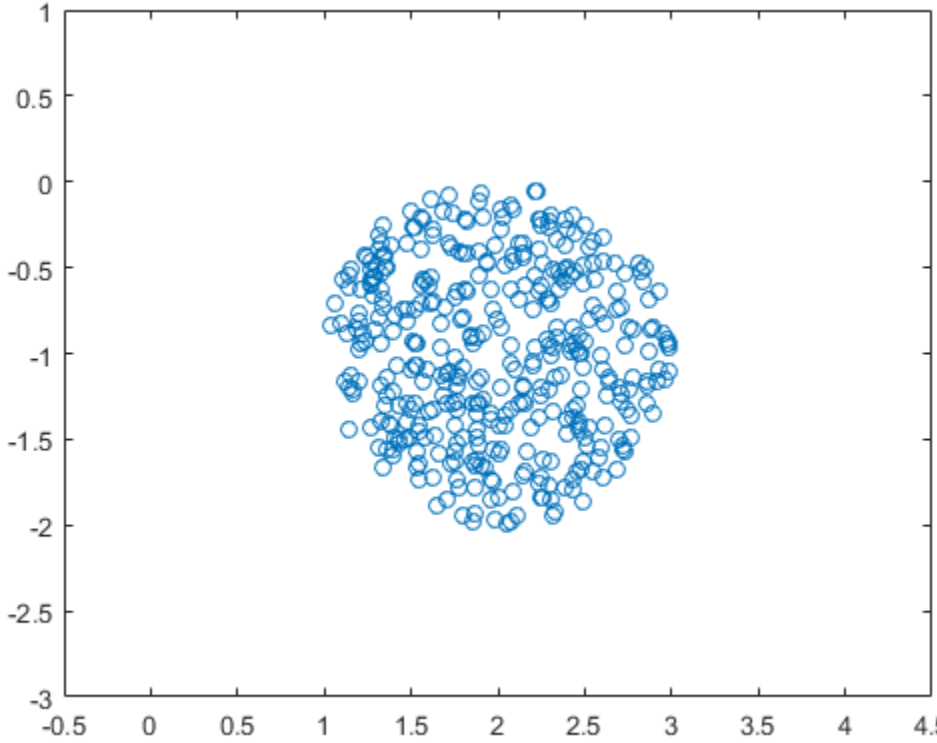
```
a =

  Uncertain complex parameter "a" with nominal value 2-1i and radius 1.
```

```
get(a)
```

```
           Name: 'a'
    NominalValue: 2.0000 - 1.0000i
           Mode: 'Radius'
         Radius: 1
     Percentage: 44.7214
    AutoSimplify: 'basic'
```

Sample the uncertain complex parameter at 400 values, and plot in the complex plane. Clearly, the samples appear to be from a disc of radius 1, centered in the complex plane at the value 2-*j*.

```
asample = usample(a,400);
plot(asample(:),'o');
xlim([-0.5 4.5]);
ylim([-3 1]);
```



Uncertain Complex Matrices

The uncertain complex matrix class, `ucomplexm`, represents the set of matrices given by the formula

$N + W_L \Delta W_R$

where N, W_L, and W_R are known matrices, and Δ is any complex matrix with $\bar{\sigma}(\Delta) \le 1$. All properties of a `ucomplexm` are can be accessed with `get` and `set`. The properties are

Properties	Meaning	Class
`Name`	Internal Name	`char`
`NominalValue`	Nominal value of element	`double`
`WL`	Left weight	`double`
`WR`	Right weight	`double`
`AutoSimplify`	`'off' | {'basic'} | 'full'`	`char`

Uncertain Complex Matrix and Weighting Matrices

Create a 4-by-3 uncertain complex matrix (`ucomplexm`), and view its properties. The simplest construction requires only a name and nominal value.

```
m = ucomplexm('m',[1 2 3; 4 5 6; 7 8 9; 10 11 12])
```

```
m =

  Uncertain complex matrix "m" with 4 rows and 3 columns.
```

```
get(m)
```

```
            Name: 'm'
    NominalValue: [4×3 double]
              WL: [4×4 double]
              WR: [3×3 double]
    AutoSimplify: 'basic'
```

The nominal value is the matrix you supply to `ucomplexm`.

```
mnom = m.NominalValue
```

```
mnom =

     1     2     3
```

```
     4     5     6
     7     8     9
    10    11    12
```

By default, the weighting matrices are the identity. For example, examine the left weighting.

```
m.WL
```

```
ans =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

Sample the uncertain matrix, and compare to the nominal value. Note the element-by-element sizes of the difference are roughly equal, indicative of the identity weighting matrices.

```
msamp = usample(m);
diff = abs(msamp-mnom)
```

```
diff =

    0.3309    0.0917    0.2881
    0.2421    0.3449    0.3917
    0.2855    0.2186    0.2915
    0.3260    0.2753    0.3816
```

Change the left and right weighting matrices, making the uncertainty larger as you move down the rows, and across the columns.

```
m.WL = diag([0.2 0.4 0.8 1.6]);
m.WR = diag([0.1 1 4]);
```

Sample the uncertain matrix again, and compare to the nominal value. Note the element-by-element sizes of the difference, and the general trend that the smallest differences are near the (1,1) element, and the largest differences are near the (4,3) element, consistent with the trend in the diagonal weighting matrices.

```
msamp = usample(m);
diff = abs(msamp-mnom)
```

```
diff =

    0.0048    0.0526    0.2735
    0.0154    0.1012    0.4898
    0.0288    0.3334    0.8555
    0.0201    0.4632    1.3783
```

See Also

`ucomplex` | `ucomplexm`

Systems with Unmodeled Dynamics

The unstructured uncertain dynamic system Control Design Block, the `udyn` object, represents completely unknown multivariable, time-varying nonlinear systems.

For practical purposes, these uncertain elements represent noncommuting symbolic variables (placeholders). All algebraic operations, such as addition, subtraction, multiplication (i.e., cascade) operate properly, and substitution (with `usubs`) is allowed. However, all of the analysis tools (e.g., `robstab`) do not handle these types of uncertain elements.

You can create a 2-by-3 `udyn` element. Check its size, and properties.

```
m = udyn('m',[2 3])
```

```
m =

  Uncertain dynamics "m" with 2 outputs and 3 inputs.
```

```
get(m)
```

```
    NominalValue: [2x3 ss]
    AutoSimplify: 'basic'
              Ts: 0
        TimeUnit: 'seconds'
       InputName: {3x1 cell}
       InputUnit: {3x1 cell}
      InputGroup: [1x1 struct]
      OutputName: {2x1 cell}
      OutputUnit: {2x1 cell}
     OutputGroup: [1x1 struct]
            Name: 'm'
           Notes: {}
        UserData: []
```

See Also

`udyn`

Related Examples

- “Uncertain LTI Dynamics Elements” on page 1-12

Uncertain Matrices

Uncertain matrices (class `umat`) are built from doubles and uncertain elements, using traditional MATLAB matrix building syntax. Uncertain matrices can be added, subtracted, multiplied, inverted, transposed, etc., resulting in uncertain matrices. The rows and columns of an uncertain matrix are referenced in the same manner that MATLAB references rows and columns of an array, using parenthesis, and integer indices. The `NominalValue` of a uncertain matrix is the result obtained when all uncertain elements are replaced with their own `NominalValue`. The uncertain elements making up a `umat` are accessible through the `Uncertainty` gateway, and the properties of each element within a `umat` can be changed directly.

Using `usubs`, specific values may be substituted for any of the uncertain elements within a `umat`. The command `usample` generates a random sample of the uncertain matrix, substituting random samples (within their ranges) for each of the uncertain elements.

The command `wcnorm` computes tight bounds on the worst-case (maximum over the uncertain elements' ranges) norm of the uncertain matrix.

Standard MATLAB numerical matrices (i.e., `double`) naturally can be viewed as uncertain matrices without any uncertainty.

Create and Manipulate Uncertain Matrices

You create uncertain matrices (umat objects) by creating uncertain parameters and using them to build matrices. You can then use uncertain matrices to build uncertain state-space models. This example shows how to create an uncertain matrix, access and change its uncertain parameters, extract elements, and perform matrix arithmetic.

For example, create two uncertain real parameters, and use them to create a 3-by-2 uncertain matrix.

```
a = ureal('a',3);
b = ureal('b',10,'Percentage',20);
M = [-a, 1/b; b, a+1/b; 1, 3]
```

```
M =

  Uncertain matrix with 3 rows and 2 columns.
  The uncertainty consists of the following blocks:
    a: Uncertain real, nominal = 3, variability = [-1,1], 2 occurrences
```

```
    b: Uncertain real, nominal = 10, variability = [-20,20]%, 3 occurrences

Type "M.NominalValue" to see the nominal value, "get(M)" to see all properties, and "M
```

Examine and Modify `umat` Properties

`M` is a `umat` object. Examine its properties using `get`.

```
get(M)
```

```
    NominalValue: [3×2 double]
     Uncertainty: [1×1 struct]
    SamplingGrid: [1×1 struct]
```

The nominal value of `M` is the matrix obtained by replacing all the uncertain elements with their nominal values.

```
M.NominalValue
```

```
ans =

   -3.0000    0.1000
   10.0000    3.1000
    1.0000    3.0000
```

The `Uncertainty` property is a structure containing the uncertain elements (the "Control Design Blocks") of `M`.

```
M.Uncertainty
```

```
ans =

  struct with fields:

    a: [1×1 ureal]
    b: [1×1 ureal]
```

```
M.Uncertainty.a
```

```
ans =

  Uncertain real parameter "a" with nominal value 3 and variability [-1,1].
```

Use the `Uncertainty` property for direct access to the uncertain elements. For example, check the `Range` of the uncertain element `a` within `M`.

```
M.Uncertainty.a.Range
```

```
ans =

     2     4
```

The range is `[2,4]` because you created the `ureal` parameter `a` with a nominal value 3 and the default uncertainty of +/- 1. Change the range to `[2.5,5]`.

```
M.Uncertainty.a.Range = [2.5,5]
```

```
M =

  Uncertain matrix with 3 rows and 2 columns.
  The uncertainty consists of the following blocks:
    a: Uncertain real, nominal = 3, variability = [-0.5,2], 2 occurrences
    b: Uncertain real, nominal = 10, variability = [-20,20]%, 3 occurrences

Type "M.NominalValue" to see the nominal value, "get(M)" to see all properties, and "M
```

This change to `a` only takes place within `M`. Verify that the variable `a` in the MATLAB workspace still has the original range.

```
a.Range
```

```
ans =

     2     4
```

You cannot combine elements that have a common internal name, but different properties. So, for example, entering `M.Uncertainty.a - a` would generate an error,

because the `realp` parameter `a` in the workspace has different properties from the element `a` in `M`.

Row and Column Referencing

You can use standard row-column referencing to extract elements from a `umat`. For example, extract a 2-by-2 selection from `M` consisting of its second and third rows.

```
Msub = M(2:3,:)
```

```
Msub =

  Uncertain matrix with 2 rows and 2 columns.
  The uncertainty consists of the following blocks:
    a: Uncertain real, nominal = 3, variability = [-0.5,2], 1 occurrences
    b: Uncertain real, nominal = 10, variability = [-20,20]%, 2 occurrences

Type "Msub.NominalValue" to see the nominal value, "get(Msub)" to see all properties, a
```

You can use single indexing only if the `umat` is a single column or row. Make a single-column selection from `M` and use single-index references to access elements of it.

```
Msing = M([2 1 2 3],2);
Msing(2)
```

```
ans =

  Uncertain matrix with 1 rows and 1 columns.
  The uncertainty consists of the following blocks:
    b: Uncertain real, nominal = 10, variability = [-20,20]%, 1 occurrences

Type "ans.NominalValue" to see the nominal value, "get(ans)" to see all properties, and
```

You can use indexing to change the value of any element of a `umat`. For example, set the (3,2) entry of `M` to an uncertain parameter `c`.

```
c = ureal('c',3,'Percentage',40);
M(3,2) = c
```

```
M =
```

```
  Uncertain matrix with 3 rows and 2 columns.
  The uncertainty consists of the following blocks:
    a: Uncertain real, nominal = 3, variability = [-0.5,2], 2 occurrences
    b: Uncertain real, nominal = 10, variability = [-20,20]%, 2 occurrences
    c: Uncertain real, nominal = 3, variability = [-40,40]%, 1 occurrences

Type "M.NominalValue" to see the nominal value, "get(M)" to see all properties, and "M
```

M now has three uncertain blocks.

Matrix Operations on `umat` Objects

You can perform many matrix operations on a `umat` object, such as matrix-multiply, transpose, and inverse. You can also combaine uncertain matrices with numeric matrices that do not have uncertainty.

For example, premultiply `M` by a `1-by-3` numeric matrix, resulting in a 1-by-2 `umat`.

```
M1 = [2 3 1]*M;
```

Verify that the first entry of `M1` is as expected, `-2*a + 3*b + 1`.

```
d = M1(1) - (-2*M.Uncertainty.a + 3*M.Uncertainty.b + 1)
```

```
d =

  Uncertain matrix with 1 rows, 1 columns, and no uncertain blocks.

Type "d.NominalValue" to see the nominal value, "get(d)" to see all properties, and "d
```

Transpose `M`, form a product, and invert it. As expected, the product of a matrix and its inverse is the identity matrix. You can verify this by sampling the result.

```
H = M.'*M;
K = inv(H);
usample(K*H,3)
```

```
ans(:,:,1) =

    1.0000    0.0000
```

```
   -0.0000    1.0000


ans(:,:,2) =

    1.0000    0.0000
   -0.0000    1.0000


ans(:,:,3) =

    1.0000    0.0000
   -0.0000    1.0000
```

Lifting a Double Matrix to `umat`

You can convert a numeric matrix to a `umat` object with no uncertain elements. Use the `umat` command to *lift* a double matrix to the `umat` class. For example:

```
Md = [1 2 3;4 5 6];
M = umat(Md)
```

```
M =

  Uncertain matrix with 2 rows, 3 columns, and no uncertain blocks.

Type "M.NominalValue" to see the nominal value, "get(M)" to see all properties, and "M
```

You can also convert higher-dimension numeric matrices to `umat`. When you do so, the software interprets the third dimension and beyond as array dimensions. For example, convert a random three-dimensional numeric array to `umat`.

```
Md = randn(4,5,6);
M = umat(Md)
```

```
M =

  6x1 array of uncertain matrices with 4 rows, 5 columns, and no uncertain blocks.

Type "M.NominalValue" to see the nominal value, "get(M)" to see all properties, and "M
```

The result is a one-dimensional array of uncertain matrices, rather than a three-dimensional uncertain array. Similarly, a four-dimensional numeric array converts to a two-dimensional array of `umat` objects.

```
Md = randn(4,5,6,7);
M = umat(Md)
```

```
M =

  6x7 array of uncertain matrices with 4 rows, 5 columns, and no uncertain blocks.

Type "M.NominalValue" to see the nominal value, "get(M)" to see all properties, and "M
```

See “Array Management for Uncertain Objects” for more information about multidimensional arrays of uncertain objects.

See Also

`umat` | `ureal`

Related Examples

- “Uncertain State-Space Models” on page 1-32

Evaluate Uncertain Elements by Substitution

You can make substitutions for uncertain elements in uncertain matrices and models using usubs. Doing so is useful for evaluating uncertain objects at particular values of the uncertain parameters, or for sampling uncertain objects at multiple parameter values.

For example, create an uncertain matrix with three uncertain parameters.

```
a = ureal('a',3);
b = ureal('b',10,'Percentage',20);
c = ureal('c',3,'Percentage',40);
M = [-a, 1/b; b, a+1/b; 1, c]
```

```
M =

  Uncertain matrix with 3 rows and 2 columns.
  The uncertainty consists of the following blocks:
    a: Uncertain real, nominal = 3, variability = [-1,1], 2 occurrences
    b: Uncertain real, nominal = 10, variability = [-20,20]%, 3 occurrences
    c: Uncertain real, nominal = 3, variability = [-40,40]%, 1 occurrences

Type "M.NominalValue" to see the nominal value, "get(M)" to see all properties, and "M
```

Substitute all instances of the uncertain real parameter `a` with the value 4. This operation results in a `umat` containing only two uncertain real parameters, `b` and `c`.

```
M2 = usubs(M,'a',4)
```

```
M2 =

  Uncertain matrix with 3 rows and 2 columns.
  The uncertainty consists of the following blocks:
    b: Uncertain real, nominal = 10, variability = [-20,20]%, 3 occurrences
    c: Uncertain real, nominal = 3, variability = [-40,40]%, 1 occurrences

Type "M2.NominalValue" to see the nominal value, "get(M2)" to see all properties, and "
```

You can replace all instances of one uncertain real parameter with another. For example, replace all insteances of `b` in `M` with the uncertain parameter `a`. The resulting `umat`

contains only the parameters `a` and `c`, and has two additional occurrences of `a`, compared to `M`.

```
M3 = usubs(M,'b',M.Uncertainty.a)
```

```
M3 =

  Uncertain matrix with 3 rows and 2 columns.
  The uncertainty consists of the following blocks:
    a: Uncertain real, nominal = 3, variability = [-1,1], 5 occurrences
    c: Uncertain real, nominal = 3, variability = [-40,40]%, 1 occurrences

Type "M3.NominalValue" to see the nominal value, "get(M3)" to see all properties, and "
```

Next, evaluate `M` at the nominal value of `a` and a random value of `b`.

```
M4 = usubs(M,'a','NominalValue','b','Random')
```

```
M4 =

  Uncertain matrix with 3 rows and 2 columns.
  The uncertainty consists of the following blocks:
    c: Uncertain real, nominal = 3, variability = [-40,40]%, 1 occurrences

Type "M4.NominalValue" to see the nominal value, "get(M4)" to see all properties, and "
```

Use the `usample` command to generate multiple random instances of `umat`, `uss`, or `ufrd` uncertain objects. See “Generate Samples of Uncertain Systems” for more information.

See Also

`ufrd` | `umat` | `usample` | `uss` | `usubs`

Related Examples

- “Substitution by usubs” on page 1-54
- “Generate Samples of Uncertain Systems” on page 1-50

Uncertain State-Space Models

Uncertain state-space (`uss`) models are linear systems with uncertain state-space matrices and/or uncertain linear dynamics. Like their numeric (i.e., not uncertain) counterpart, the`ss` model object, you can build them from state-space matrices using the `ss` command. When one or more of the state-space matrices contain uncertain elements (uncertain Control Design Blocks), the result is a `uss` model object.

Combining uncertain systems with other uncertain systems (for example, using model arithmetic, `connect`, or `feedback`) usually results in an uncertain system. You can also combine numeric systems with uncertain systems. Usually the result is an uncertain system. The nominal value of an uncertain system is a `ss` model object.

In the example below, the `A`, `B` and `C` matrices are made up of uncertain real parameters. Packing them together with the `ss` command results in a continuous-time uncertain system.

Create Uncertain State-Space (uss) Model

To create an uncertain state-space model, you first use control design blocks to create uncertain elements. Then, use the elements to specify the state-space matrices of the system.

For instance, create three uncertain real parameters and build state-spaces matrices from them.

```
p1 = ureal('p1',10,'Percentage',50);
p2 = ureal('p2',3,'PlusMinus',[-.5 1.2]);
p3 = ureal('p3',0);

A = [-p1 p2; 0 -p1];
B = [-p2; p2+p3];
C = [1 0; 1 1-p3];
D = [0; 0];
```

The matrices constructed with uncertain parameters, `A`, `B`, and `C`, are `umat` uncertain matrix objects. Using them as inputs to the `ss` command results in a 2-output, 1-input, 2-state uncertain system.

```
sys = ss(A,B,C,D)
```

```
sys =

  Uncertain continuous-time state-space model with 2 outputs, 1 inputs, 2 states.
  The model uncertainty consists of the following blocks:
    p1: Uncertain real, nominal = 10, variability = [-50,50]%, 2 occurrences
    p2: Uncertain real, nominal = 3, variability = [-0.5,1.2], 2 occurrences
    p3: Uncertain real, nominal = 0, variability = [-1,1], 2 occurrences

Type "sys.NominalValue" to see the nominal value, "get(sys)" to see all properties, and
```

The display shows that the system includes the three uncertain parameters.

Properties of uss Objects

`uss` models, like all model objects, include properties that store dynamics and model metadata. View the properties of an uncertain state-space model.

```
p1 = ureal('p1',10,'Percentage',50);
p2 = ureal('p2',3,'PlusMinus',[-.5 1.2]);
p3 = ureal('p3',0);
A = [-p1 p2; 0 -p1];
B = [-p2; p2+p3];
C = [1 0; 1 1-p3];
D = [0; 0];
sys = ss(A,B,C,D);     % create uss model

get(sys)
```

```
                A: [2×2 umat]
                B: [2×1 umat]
                C: [2×2 umat]
                D: [2×1 double]
                E: []
        StateName: {2×1 cell}
        StateUnit: {2×1 cell}
     NominalValue: [2×1 ss]
      Uncertainty: [1×1 struct]
    InternalDelay: [0×1 double]
       InputDelay: 0
      OutputDelay: [2×1 double]
               Ts: 0
         TimeUnit: 'seconds'
```

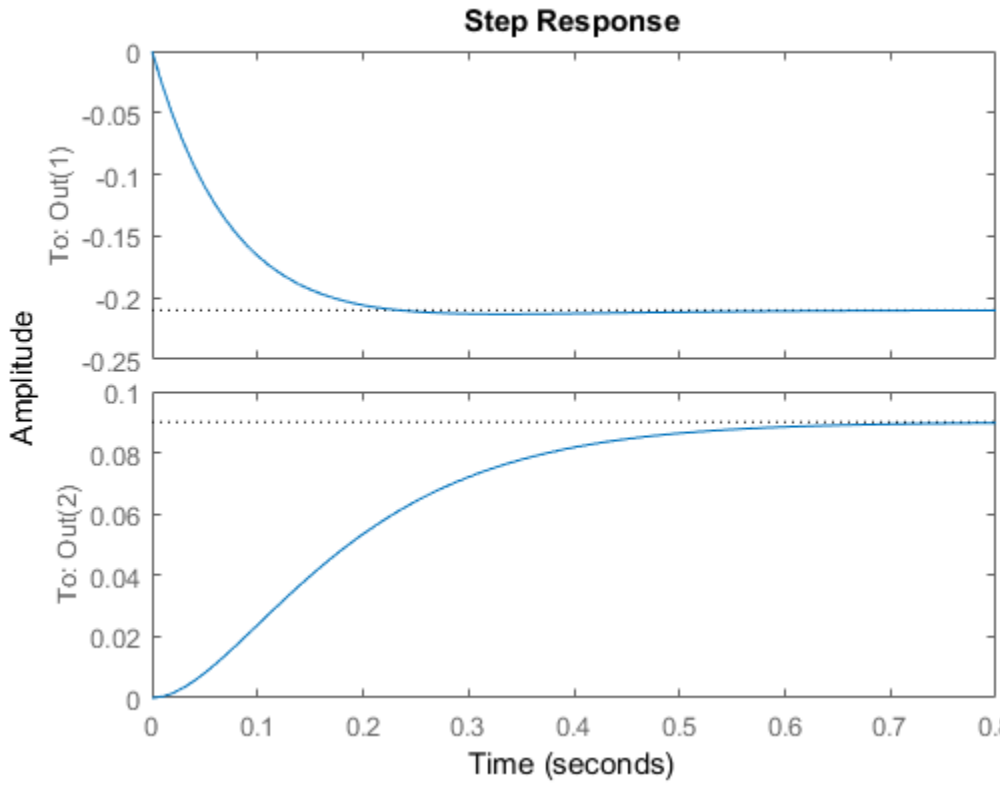
```
        InputName: {''}
        InputUnit: {''}
       InputGroup: [1×1 struct]
       OutputName: {2×1 cell}
       OutputUnit: {2×1 cell}
      OutputGroup: [1×1 struct]
             Name: ''
            Notes: {}
         UserData: []
     SamplingGrid: [1×1 struct]
```

Most of the properties behave similarly to how they behave for `ss` model objects. The `NominalValue` property is itslef an `ss` model object. You can therefore analyze the nominal value as you would any state-space model. For instance, compute the poles and step response of the nominal system.

```
pole(sys.NominalValue)
```

```
ans =

   -10
   -10
```

```
step(sys.NominalValue)
```



As with the uncertain matrices (`umat`), the `Uncertainty` property is a structure containing the uncertain elements. You can use this property for direct access to the uncertain elements. For instance, check the `Range` of the uncertain element named `p2` within `sys`.

```
sys.Uncertainty.p2.Range
```

```
ans =

    2.5000    4.2000
```

Change the uncertainty range of `p2` within `sys`.

```
sys.Uncertainty.p2.Range = [2 4];
```

This command only changes the range of the parameter called `p2` in `sys`. It does not change the variable `p2` in the MATLAB workspace.

```
p2.Range
```

```
ans =

    2.5000    4.2000
```

Lifting a ss to a uss

A not-uncertain state space object may be interpreted as an uncertain state space object that has no dependence on uncertain elements. Use the `uss` command to "lift" a `ss` to the `uss` class.

```
sys = rss(3,2,1);
usys = uss(sys)
USS: 3 States, 2 Outputs, 1 Input, Continuous System
```

Arrays of `ss` objects can also be lifted. See "Array Management for Uncertain Objects" on page 1-57 for more information about how arrays of uncertain objects are handled.

See Also

`ss` | `umat` | `uss`

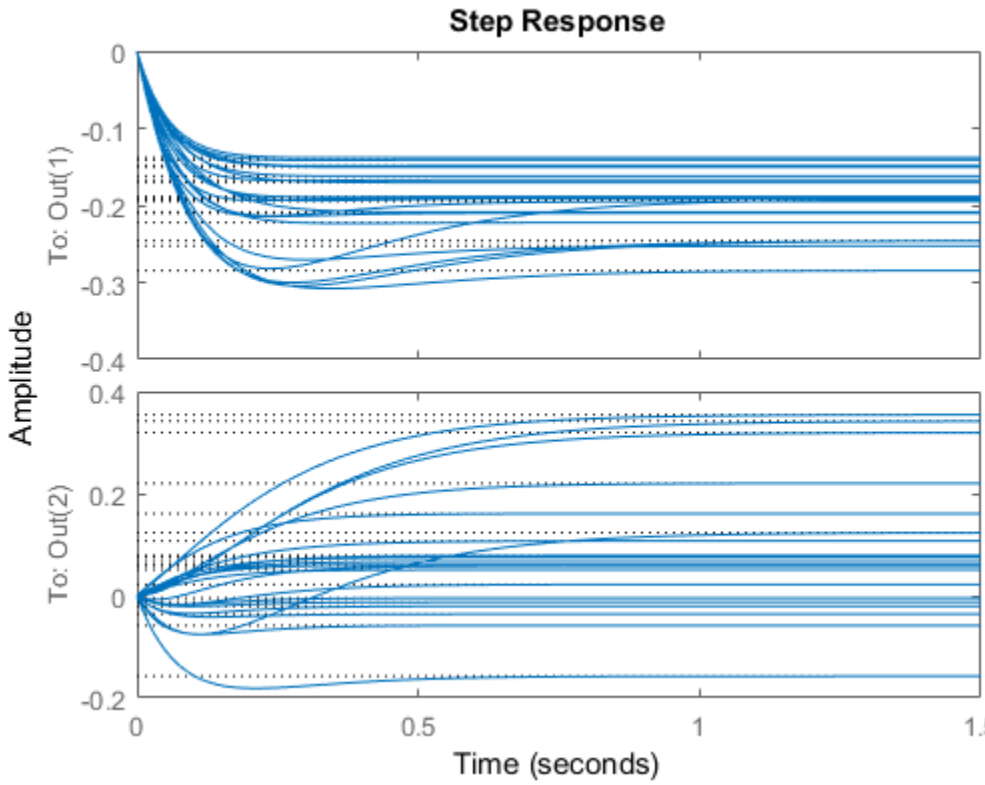
Related Examples

- "Create and Manipulate Uncertain Matrices" on page 1-23
- "Uncertain Model Interconnections" on page 1-39

Sample Uncertain Systems

The command `usample` randomly samples the uncertain system at a specified number of points. Randomly sample an uncertain system at 20 points in its modeled uncertainty range. This gives a 20-by-1 `ss` array. Consequently, all analysis tools from Control System Toolbox™ are available.

```
p1 = ureal('p1',10,'Percentage',50);
p2 = ureal('p2',3,'PlusMinus',[-.5 1.2]);
p3 = ureal('p3',0);
A = [-p1 p2; 0 -p1];
B = [-p2; p2+p3];
C = [1 0; 1 1-p3];
D = [0; 0];

sys = ss(A,B,C,D) % Create uncertain state-space model


sys =

  Uncertain continuous-time state-space model with 2 outputs, 1 inputs, 2 states.
  The model uncertainty consists of the following blocks:
    p1: Uncertain real, nominal = 10, variability = [-50,50]%, 2 occurrences
    p2: Uncertain real, nominal = 3, variability = [-0.5,1.2], 2 occurrences
    p3: Uncertain real, nominal = 0, variability = [-1,1], 2 occurrences

Type "sys.NominalValue" to see the nominal value, "get(sys)" to see all properties, and


manysys = usample(sys,20);
size(manysys)

20x1 array of state-space models.
Each model has 2 outputs, 1 inputs, and 2 states.

stepplot(manysys)
```



The command `stepplot` can be called directly on a `uss` object. The default behavior samples the `uss` object at 20 instances, and plots the step responses of these 20 models, as well as the nominal value.

The same features are available for other analysis commands such as `bodeplot`, `bodemag`, `impulse`, and `nyquist`.

See Also

`usample` | `uss`

Related Examples

- “Generate Samples of Uncertain Systems” on page 1-50

Uncertain Model Interconnections

Feedback Around an Uncertain Plant

It is possible to form interconnections of `uss` objects. A common example is to form the feedback interconnection of a given controller with an uncertain plant.

First create the uncertain plant. Start with two uncertain real parameters.

```
gamma = ureal('gamma',4);
tau = ureal('tau',.5,'Percentage',30);
```

Next, create an unmodeled dynamics element, `delta`, and a first-order weighting function, whose DC value is 0.2, high-frequency gain is 10, and whose crossover frequency is 8 rad/sec.

```
delta = ultidyn('delta',[1 1],'SampleStateDimension',5);
W = makeweight(0.2,6,6);
```

Finally, create the uncertain plant consisting of the uncertain parameters and the unmodeled dynamics.

```
P = tf(gamma,[tau 1])*(1+W*delta);
```

You can create an integral controller based on nominal plant parameters. Nominally the closed-loop system will have damping ratio of 0.707 and time constant of `2*tau`.

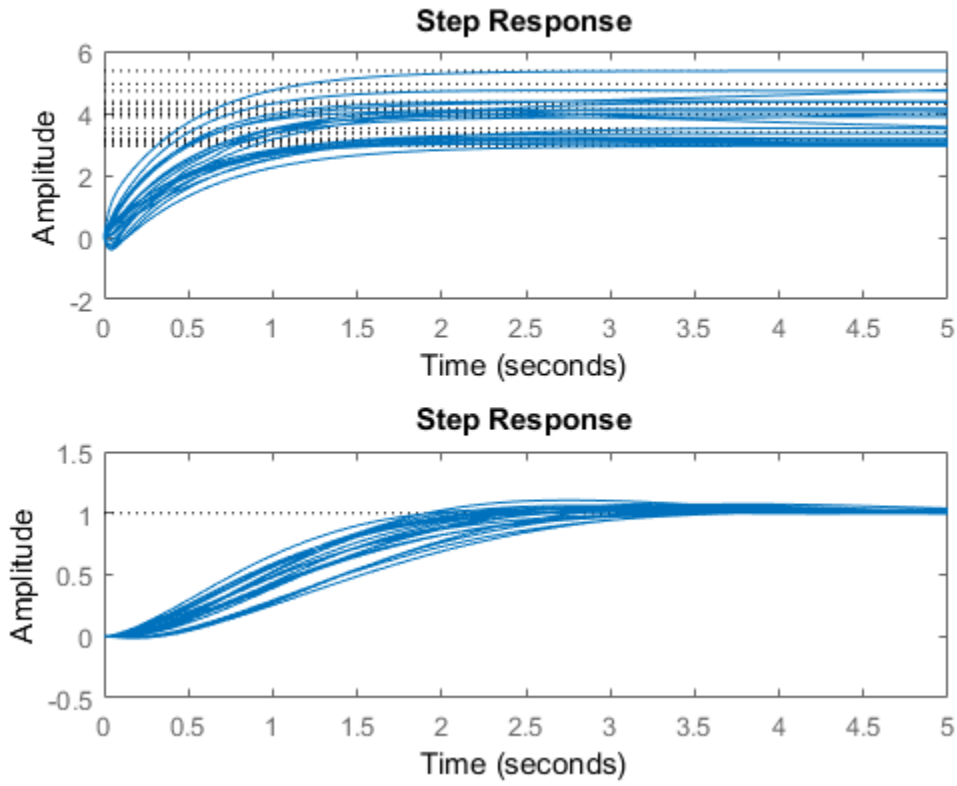
```
KI = 1/(2*tau.Nominal*gamma.Nominal);
C = tf(KI,[1 0]);
```

Create the uncertain closed-loop system using the `feedback` command.

```
CLP = feedback(P*C,1);
```

Plot samples of the open-loop and closed-loop step responses. As expected the integral controller reduces the variability in the low frequency response.

```
subplot(2,1,1);
stepplot(P,5)
subplot(2,1,2);
stepplot(CLP,5)
```



Basic Model Interconnections

All the model arithmetic and model-interconnection commands of Control System Toolbox software work with uncertain models. These include:

- `connect`
- `feedback`
- `series`
- `parallel`
- `append`
- `blkdiag`

- `lft`
- `stack`

For more information about model interconnections, see “Model Interconnection” in the Control System Toolbox documentation.

Create Uncertain Frequency Response Data Models

Uncertain frequency responses (`ufrd`) arise naturally when computing the frequency response of an uncertain state-space (`uss`). They also arise when frequency response data (in an `frd` model object) is combined (added, multiplied, concatenated, etc.) to an uncertain matrix (`umat`).

The most common manner in which a `ufrd` arises is taking the frequency response of a `uss`. To do this, use the `ufrd` command.

Construct an uncertain state-space model `sys`.

```
p1 = ureal('p1',10,'pe',50);
p2 = ureal('p2',3,'plusm',[-.5 1.2]);
p3 = ureal('p3',0);
A = [-p1 p2;0 -p1];
B = [-p2;p2+p3];
C = [1 0;1 1-p3];
D = [0;0];
sys = ss(A,B,C,D)
```

```
sys =

  Uncertain continuous-time state-space model with 2 outputs, 1 inputs, 2 states.
  The model uncertainty consists of the following blocks:
    p1: Uncertain real, nominal = 10, variability = [-50,50]%, 2 occurrences
    p2: Uncertain real, nominal = 3, variability = [-0.5,1.2], 2 occurrences
    p3: Uncertain real, nominal = 0, variability = [-1,1], 2 occurrences

Type "sys.NominalValue" to see the nominal value, "get(sys)" to see all properties, and
```

Compute the uncertain frequency response of the uncertain system. Use the `ufrd` command, along with a frequency grid containing 100 points. The result is an uncertain frequency response data object, referred to as a `ufrd` model.

```
sysg = ufrd(sys,logspace(-2,2,100))
```

```
  Uncertain continuous-time FRD model with 2 outputs, 1 inputs, 100 frequency points.
    p1: Uncertain real, nominal = 10, variability = [-50,50]%, 2 occurrences
    p2: Uncertain real, nominal = 3, variability = [-0.5,1.2], 2 occurrences
    p3: Uncertain real, nominal = 0, variability = [-1,1], 2 occurrences

Type "sysg.NominalValue" to see the nominal value, "get(sysg)" to see all properties, a
```

Properties of ufrd Objects

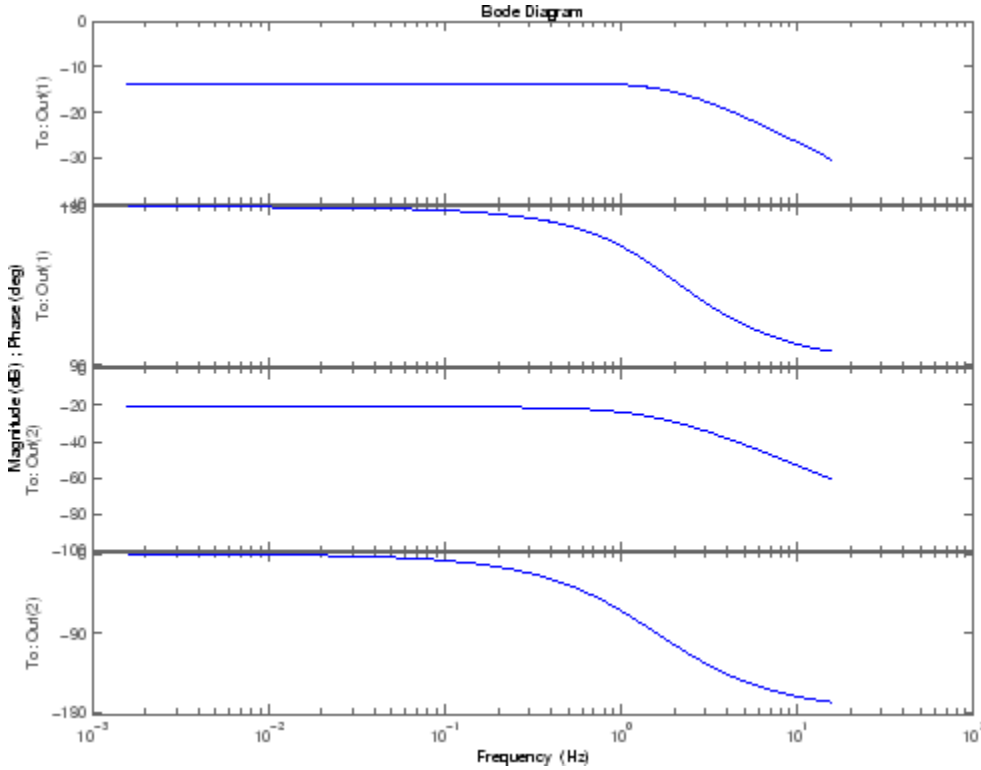
View the properties with the `get` command.

```
get(sysg)
        Frequency: [100x1 double]
    FrequencyUnit: 'rad/TimeUnit'
     ResponseData: [4-D umat]
     NominalValue: [2x1 frd]
      Uncertainty: [1x1 struct]
       InputDelay: 0
      OutputDelay: [2x1 double]
               Ts: 0
         TimeUnit: 'seconds'
        InputName: {''}
        InputUnit: {''}
       InputGroup: [1x1 struct]
       OutputName: {2x1 cell}
       OutputUnit: {2x1 cell}
      OutputGroup: [1x1 struct]
             Name: ''
            Notes: {}
         UserData: []
```

The properties `ResponseData` and `Frequency` behave in exactly the same manner as Control System Toolbox `frd` objects, except that `ResponseData` is a `umat`. The properties `InputName, OutputName, InputGroup` and `OutputGroup` behave in exactly the same manner as all of the Control System Toolbox objects (`ss`, `zpk`, `tf` and `frd`).

The `NominalValue` is a Control System Toolbox `frd` object, and hence all methods for frd objects are available. For instance, plot the Bode response of the nominal system.

```
bode(sysg.nom)
```



Just as with the `umat` and `uss` classes, the `Uncertainty` property is a structure containing the uncertain elements. Direct access to the elements is facilitated through `Uncertainty`. Change the nominal value of the uncertain element named `'p1'` within `sysg` to 14, and replot the Bode plot of the (new) nominal system.

```
sysg.unc.p1.nom = 14
UFRD: 2 Outputs, 1 Input, Continuous System, 100 Frequency points
  p1: real, nominal = 14, variability = [-50  50]%, 2 occurrences
  p2: real, nominal = 3, variability = [-0.5  1.2], 2 occurrences
  p3: real, nominal = 0, variability = [-1  1], 2 occurrences
```

Lifting an frd to a ufrd

A not-uncertain frequency response object may be interpreted as an uncertain frequency response object that has no dependence on uncertain elements. Use the `ufrd` command to “lift” an `frd` object to the `ufrd` class.

```
sys = rss(3,2,1);
sysg = frd(sys,logspace(-2,2,100));
usysg = ufrd(sysg)
UFRD: 2 Outputs, 1 Input, Continuous System, 100 Frequency points
```

Arrays of `frd` objects can also be lifted. See “Array Management for Uncertain Objects” on page 1-57 for more information about how arrays of uncertain objects are handled.

See Also

`ufrd`

Simplifying Representation of Uncertain Objects

A minimal realization of the transfer function matrix

$$H(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{4}{s+1} \\ \frac{3}{s+1} & \frac{6}{s+1} \end{bmatrix}$$

has only 1 state, obvious from the decomposition

$$H(s) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \frac{1}{s+1} \begin{bmatrix} 1 & 2 \end{bmatrix}.$$

However, a “natural” construction, formed by

```
sys11 = ss(tf(2,[1 1]));
sys12 = ss(tf(4,[1 1]));
sys21 = ss(tf(3,[1 1]));
sys22 = ss(tf(6,[1 1]));
sys = [sys11 sys12;sys21 sys22]
a =
       x1  x2  x3  x4
   x1  -1   0   0   0
   x2   0  -1   0   0
   x3   0   0  -1   0
   x4   0   0   0  -1
b =
       u1  u2
   x1   2   0
   x2   0   2
   x3   2   0
   x4   0   2
c =
        x1   x2   x3   x4
   y1    1    2    0    0
   y2    0    0  1.5    3
d =
       u1  u2
   y1   0   0
   y2   0   0
```

```
Continuous-time model
```

has four states, and is nonminimal.

In the same manner, the internal representation of uncertain objects built up from uncertain elements can become nonminimal, depending on the sequence of operations in their construction. The command `simplify` employs ad-hoc simplification and reduction schemes to reduce the complexity of the representation of uncertain objects. There are three levels of simplification: off, basic and full. Each uncertain element has an `AutoSimplify` property whose value is either `'off'`, `'basic'` or `'full'`. The default value is `'basic'`.

After (nearly) every operation, the command `simplify` is automatically run on the uncertain object, cycling through all of the uncertain elements, and attempting to simplify (without error) the representation of the effect of that uncertain object. The `AutoSimplify` property of each element dictates the types of computations that are performed. In the `'off'` case, no simplification is even attempted. In `'basic'`, fairly simple schemes to detect and eliminate nonminimal representations are used. Finally, in `'full'`, numerical based methods similar to truncated balanced realizations are used, with a very tight tolerance to minimize error.

Effect of the Autosimplify Property

Create an uncertain real parameter, view the `AutoSimplify` property of `a`, and then create a 1-by-2 `umat`, both of whose entries involve the uncertain parameter.

```
a = ureal('a',4);
a.AutoSimplify
ans =
basic
m1 = [a+4 6*a]
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 1 occurrence
```

Note that although the uncertain real parameter a appears in both (two) entries of the matrix, the resulting uncertain matrix `m1` only depends on “1 occurrence” of `a`.

Set the `AutoSimplify` property of `a` to `'off'` (from `'basic'`). Recreate the 1-by-2 `umat`. Now note that the resulting uncertain matrix `m2` depends on “2 occurrences” of `a`.

```
a.AutoSimplify = 'off';
m2 = [a+4 6*a]
```

```
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 2 occurrences
```

The 'basic' level of autosimplification often detects (and simplifies) duplication created by linear terms in the various entries. Higher order (quadratic, bilinear, etc.) duplication is often not detected by the 'basic' autosimplify level.

For example, reset the AutoSimplify property of a to 'basic' (from 'off'). Create an uncertain real parameter, and a 1-by-2 umat, both of whose entries involve the square of the uncertain parameter.

```
a.AutoSimplify = 'basic';
m3 = [a*(a+4) 6*a*a]
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 4 occurrences
```

Note that the resulting uncertain matrix m3 depends on "4 occurrences" of a.

Set the AutoSimplify property of a to 'full' (from 'basic'). Recreate the 1-by-2 umat. Now note that the resulting uncertain matrix m4 depends on "2 occurrences" of a.

```
a.AutoSimplify = 'full';
m4 = [a*(a+4) 6*a*a]
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 2 occurrences
```

Although m4 has a less complex representation (2 occurrences of a rather than 4 as in m3), some numerical variations are seen when both uncertain objects are evaluated at (say) 0.

```
usubs(m3,'a',0)
ans =
     0     0
usubs(m4,'a',0)
ans =
  1.0e-015 *
   -0.4441         0
```

Small numerical differences are also noted at other evaluation points. The example below shows the differences encountered evaluating at a equal to 1.

```
usubs(m3,'a',1)
ans =
     5     6
```

```
usubs(m4,'a',1)
ans =
    5.0000    6.0000
```

Direct Use of simplify

The `simplify` command can be used to override all uncertain element's `AutoSimplify` property. The first input to the `simplify` command is an uncertain object. The second input is the desired reduction technique, which can either `'basic'` or `'full'`.

Again create an uncertain real parameter, and a 1-by-2 `umat`, both of whose entries involve the square of the uncertain parameter. Set the `AutoSimplify` property of `a` to `'basic'`.

```
a.AutoSimplify = 'basic';
m3 = [a*(a+4) 6*a*a]
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 4 occurrences
```

Note that the resulting uncertain matrix `m3` depends on four occurrences of `a`.

The `simplify` command can be used to perform a `'full'` reduction on the resulting `umat`.

```
m4 = simplify(m3,'full')
UMAT: 1 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 2 occurrences
```

The resulting uncertain matrix `m4` depends on only two occurrences of `a` after the reduction.

See Also

`simplify`

Related Examples

- “Introduction to Uncertain Elements” on page 1-2
- “Decomposing Uncertain Objects” on page 1-70

Generate Samples of Uncertain Systems

Use the `usample` function to randomly sample an uncertain model, returning one or more non-uncertain instances of the uncertain model.

Generating One Sample

If `A` is an uncertain object, then `usample(A)` generates a single sample of `A`.

For example, a sample of a `ureal` is a scalar `double`.

```
A = ureal('A',6);
B = usample(A)
B =
    5.7298
```

Create a 1-by-3 `umat` with `A` and an uncertain complex parameter `C`. A single sample of this `umat` is a 1-by-3 double.

```
C = ucomplex('C',2+6j);
M = [A C A*A];
usample(M)
ans =
   5.9785             1.4375 + 6.0290i  35.7428
```

Generating Many Samples

If `A` is an uncertain object, then `usample(A,N)` generates `N` samples of `A`.

For example, 20 samples of a `ureal` gives a 1-by-1-20 `double` array.

```
B = usample(A,20);
size(B)
ans =
     1     1    20
```

Similarly, 30 samples of the 1-by-3 `umat` `M` yields a 1-by-3-by-30 array.

```
size(usample(M,30))
ans =
```

```
     1     3    30
```

See “Create Arrays with usample” on page 1-63 for more information on sampling uncertain objects.

Sampling ultidyn Elements

When sampling an `ultidyn` element or an uncertain object that contains a `ultidyn` element, the result is always a state-space (`ss`) object. The property `SampleStateDimension` of the `ultidyn` class determines the state dimension of the samples.

Create a 1-by-1, gain bounded `ultidyn` object with gain bound 3. Verify that the default state dimension for samples is 1.

```
del = ultidyn('del',[1 1],'Bound',3);
del.SampleStateDimension
```

```
ans =

     1
```

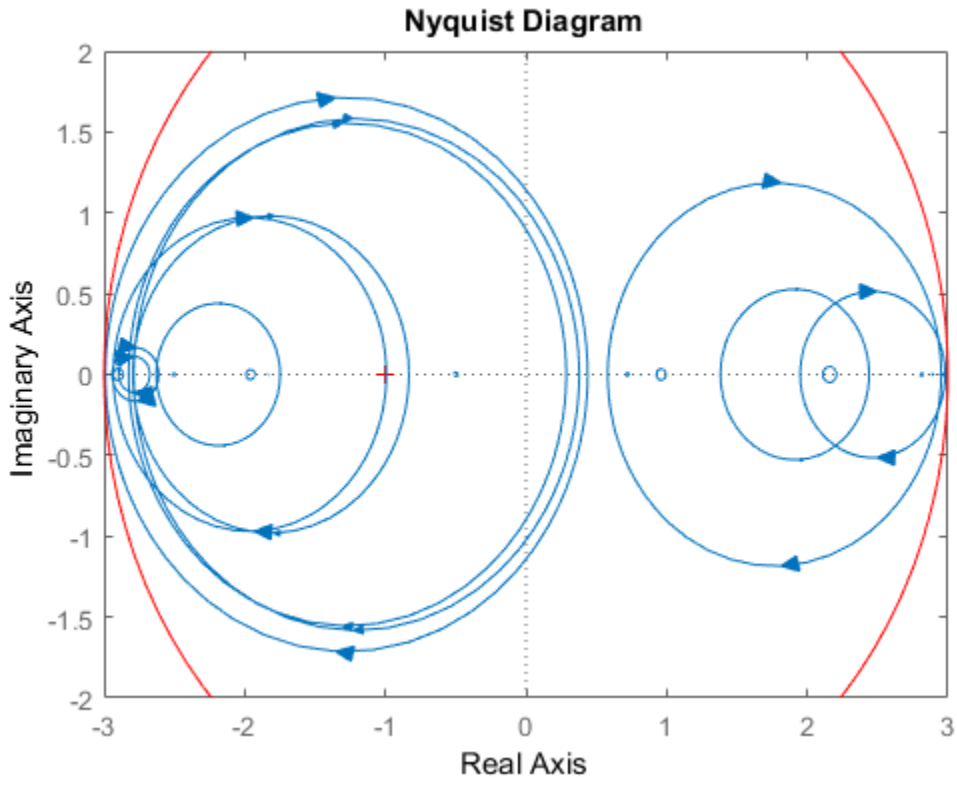
Sample the uncertain element at 30 points. Verify that this creates a 30-by-1 `ss` array of 1-input, 1-output, 1-state systems.

```
delS = usample(del,30);
size(delS)
```

```
30x1 array of state-space models.
Each model has 1 outputs, 1 inputs, and 1 states.
```

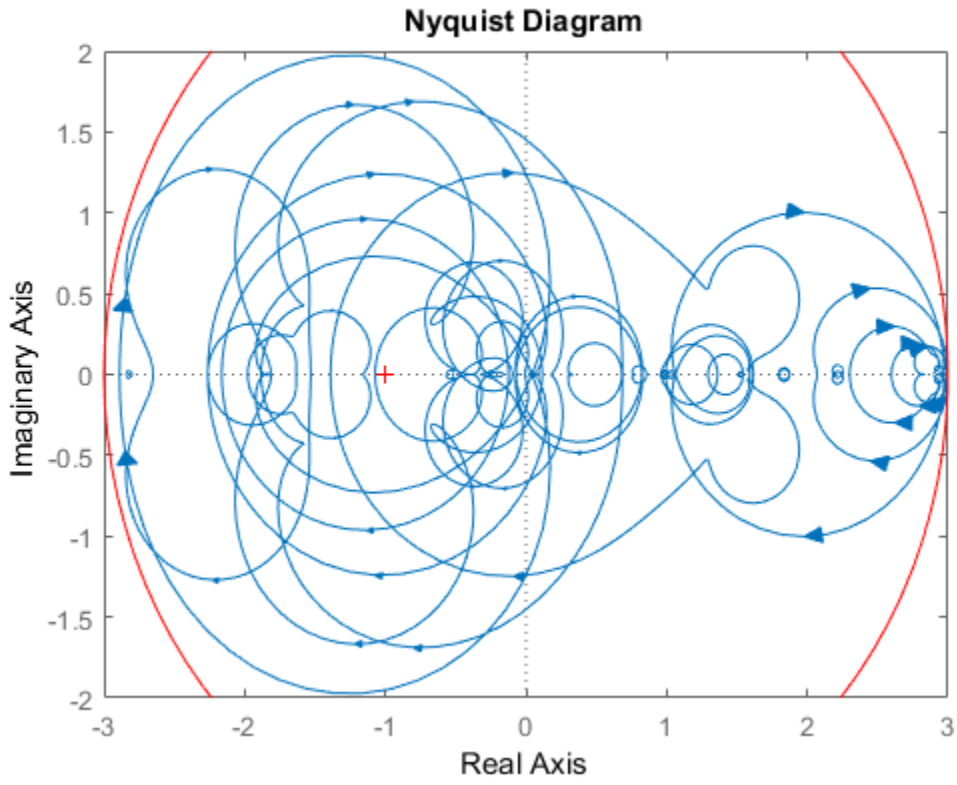
Plot the Nyquist plot of these samples and add a disk of radius 3. Note that the gain bound is satisfied and that the Nyquist plots are all circles, indicative of 1st order systems.

```
nyquist(delS)
hold on;
theta = linspace(-pi,pi);
plot(del.Bound*exp(sqrt(-1)*theta),'r');
hold off;
```



Change `SampleStateDimension` to 4, and repeat entire procedure. The Nyquist plots satisfy the gain bound and as expected are more complex than the circles found in the 1st-order sampling.

```
del.SampleStateDimension = 4;
delS = usample(del,30);
nyquist(delS)
hold on;
theta = linspace(-pi,pi);
plot(del.Bound*exp(sqrt(-1)*theta),'r');
hold off;
```



See Also

`usample` | `usubs`

Related Examples

- “Evaluate Uncertain Elements by Substitution” on page 1-30

Substitution by usubs

If an uncertain object (`umat`, `uss`, `ufrd`) has many uncertain parameters, it is often necessary to freeze some, but not all, of the uncertain parameters to specific values. The `usubs` command accomplishes this, and also allows more complicated substitutions for an element.

`usubs` accepts a list of element names, and respective values to substitute for them. You can create three uncertain real parameters and use them to create a 2-by-2 uncertain matrix `A`.

```
delta = ureal('delta',2);
eta = ureal('eta',6);
rho = ureal('rho',-1);
A = [3+delta+eta delta/eta;7+rho rho+delta*eta]
```

```
A =

  Uncertain matrix with 2 rows and 2 columns.
  The uncertainty consists of the following blocks:
    delta: Uncertain real, nominal = 2, variability = [-1,1], 2 occurrences
    eta: Uncertain real, nominal = 6, variability = [-1,1], 3 occurrences
    rho: Uncertain real, nominal = -1, variability = [-1,1], 1 occurrences

Type "A.NominalValue" to see the nominal value, "get(A)" to see all properties, and "A.
```

Use `usubs` to substitute the uncertain element named `delta` in `A` with the value 2.3, leaving all other uncertain elements intact. Note that the result, `B`, is an uncertain matrix with dependence only on `eta` and `rho`.

```
B = usubs(A,'delta',2.3)
```

```
B =

  Uncertain matrix with 2 rows and 2 columns.
  The uncertainty consists of the following blocks:
    eta: Uncertain real, nominal = 6, variability = [-1,1], 3 occurrences
    rho: Uncertain real, nominal = -1, variability = [-1,1], 1 occurrences

Type "B.NominalValue" to see the nominal value, "get(B)" to see all properties, and "B.
```

To set multiple elements, list individually, or group the values in a data structure. For instance, the following are the same.

```
B1 = usubs(A,'delta',2.3,'eta',A.Uncertainty.rho);
S.delta = 2.3;
S.eta = A.Uncertainty.rho;
B2 = usubs(A,S);
```

In each case, `delta` is replaced by 2.3, and `eta` is replaced by `A.Uncertainty.rho`.

Any superfluous substitution requests are ignored. Hence, the following returns an uncertain model that is the same as `A`.

```
B4 = usubs(A,'fred',5);
```

Specifying the Substitution with Structures

An alternative syntax for `usubs` is to specify the substituted values in a structure, whose fieldnames are the names of the elements being substituted with values.

Create a structure `NV` with 2 fields, `delta` and `eta`. Set the values of these fields to be the desired substituted values. Then perform the substitution with `usubs`.

```
NV.delta = 2.3;
NV.eta = A.Uncertainty.rho;
B6 = usubs(A,NV);
```

Here, `B6` is the same as `B1` and `B2` above. Again, any superfluous fields are ignored. Therefore, adding an additional field `gamma` to `NV`, and substituting does not alter the result.

```
NV.gamma = 0;
B7 = usubs(A,NV);
```

Here, `B7` is the same as `B6`.

Analysis commands such as `wcgain`, `robstab` and `usample` all return substitutable values in this structure format. More discussion can be found in “Create Arrays with usubs” on page 1-65.

Nominal and Random Values

If the replacement value is `'Nominal'` (or a shortened version such as `'Nom'`), then the listed element are replaced with their nominal values. Therefore

```
B8 = usubs(A,fieldnames(A.Uncertainty),'nom')
```

```
B8 =

   11.0000    0.3333
    6.0000   11.0000

B9 = A.NominalValue

B9 =

   11.0000    0.3333
    6.0000   11.0000
```

are the same. It is possible to only set some of the elements to `NominalValues`, and would be the typical use of `usubs` with the `'nominal'` argument.

Within `A`, set `eta` to its nominal value, `delta` to a random value (within its range) and `rho` to a specific value, say 6.5

```
B10 = usubs(A,'eta','nom','delta','rand','rho',6.5)

B10 =

   10.5183    0.2531
   13.5000   15.6100
```

Unfortunately, the `'nominal'` and `'Random'` specifiers may not be used in the structure format. However, explicitly setting a field of the structure to an element's nominal value, and then following (or preceeding) the call to `usubs` with a call to `usample` (to generate the random samples) is acceptable, and achieves the same effect.

See Also

`usample` | `usubs`

Related Examples

- “Sample Uncertain Systems” on page 1-37
- “Evaluate Uncertain Elements by Substitution” on page 1-30

Array Management for Uncertain Objects

All of the uncertain system classes (`uss`, `ufrd`) may be multidimensional arrays. This is intended to provide the same functionality as the LTI-arrays of the Control System Toolbox software. The command `size` returns a row vector with the sizes of all dimensions.

The first two dimensions correspond to the outputs and inputs of the system. Any dimensions beyond are referred to as the *array dimensions*. Hence, if `szM = size(M)`, then `szM(3:end)` are sizes of the array dimensions of `M`.

For these types of objects, it is clear that the first two dimensions (system output and input) are interpreted differently from the 3rd, 4th, 5th and higher dimensions (which often model parametrized variability in the system input/output behavior).

`umat` objects are treated in the same manner. The first two dimensions are the rows and columns of the uncertain matrix. Any dimensions beyond are the *array dimensions*.

Reference Into Arrays

Suppose `M` is a `umat`, `uss` or `ufrd`, and that `Yidx` and `Uidx` are vectors of integers. Then

```
M(Yidx,Uidx)
```

selects the outputs (rows) referred to by `Yidx` and the inputs (columns) referred to by `Uidx`, preserving all of the array dimensions. For example, if `size(M)` equals `[4 5 3 6 7]`, then (for example) the size of `M([4 2],[1 2 4])` is `[2 3 3 6 7]`.

If `size(M,1)==1` or `size(M,2)==1`, then single indexing on the inputs or outputs (rows or columns) is allowed. If `Sidx` is a vector of integers, then `M(Sidx)` selects the corresponding elements. All array dimensions are preserved.

If there are `K` array dimensions, and `idx1, idx2, ..., idxK` are vectors of integers, then

```
G = M(Yidx,Uidx,idx1,idx2,...,idxK)
```

selects the outputs and inputs referred to by `Yidx` and `Uidx`, respectively, and selects from each array dimension the “slices” referred to by the `idx1, idx2,..., idxK` index vectors. Consequently, `size(G,1)` equals `length(Yidx), size(G,2)`

equals `length(Uidx), size(G,3)` equals `length(idx1), size(G,4)` equals `length(idx2)`, and `size(G,K+2)` equals `length(idxK)`.

If `M` has `K` array dimensions, and less than `K` index vectors are used in doing the array referencing, then the MATLAB convention for single indexing is followed. For instance, suppose `size(M)` equals `[3 4 6 5 7 4]`. The expression

```
G = M([1 3],[1 4],[2 3 4],[5 3 1],[8 10 12 2 4 20 18])
```

is valid. The result has `size(G)` equals `[2 2 3 3 7]` . The last index vector `[8 10 12 2 4 20 18]` is used to reference into the 7-by-4 array, preserving the order dictated by MATLAB single indexing (e.g., the 10th element of a 7-by-4 array is the element in the (3,2) position in the array).

Note that if `M` has either one output (row) or one input (column), *and* `M` has array dimensions, then it is not allowable to combine single indexing in the output/input dimensions along with indexing in the array dimensions. This will result in an ambiguity in how to interpret the second index vector in the expression (i.e., "does it correspond to the input/output reference, or does it correspond to the first array dimension?").

Related Examples

- "Model Arrays"

Create Arrays with stack and cat Functions

An easy manner to create an array is with `stack`. Create a [4-by-1] `umat` array by stacking four 1-by-3 `umat` objects with the `stack` command. The first argument of `stack` specifies in which array dimension the stacking occurs. In the example below, the stacking is done is the 1st array dimension, hence the result is a 1-by-3-by-4-by-1 `umat`, referred to as a 4-by-1 `umat` array.

```
a = ureal('a',4);
b = ureal('b',2);
M = stack(1,[a b 1],[-a -b 4+a],[4 5 6],[a 0 0])
UMAT: 1 Rows, 3 Columns [array, 4 x 1]
  a: real, nominal = 4, variability = [-1  1], 1 occurrence
  b: real, nominal = 2, variability = [-1  1], 1 occurrence
size(M)
ans =
     1     3     4
arraysize(M)
ans =
     4     1
```

Check that result is valid. Use referencing to access parts of the [4-by-1] `umat` array and compare to the expected values. The first 4 examples should all be arrays full of 0 (zeros). The last two should be the value 5, and the uncertain real parameter `a`, respectively.

```
simplify(M(:,:,1) - [a b 1])
ans =
     0     0     0
simplify(M(:,:,2) - [-a -b 4+a])
ans =
     0     0     0
simplify(M(:,:,3) - [4 5 6])
ans =
     0     0     0
simplify(M(:,:,4) - [a 0 0])
ans =
     0     0     0
simplify(M(1,2,3))  % should be 5
ans =
     5
simplify(M(1,3,2)-4)
Uncertain Real Parameter: Name a, NominalValue 4, variability = [-1  1]
```

You can create a random 1-by-3-by-4 double matrix and stack this with M along the second array dimension, creating a 1-by-3-by-4-by-2 umat.

```
N = randn(1,3,4);
M2 = stack(2,M,N);
size(M2)
ans =
     1     3     4     2
arraysize(M2)
ans =
     4     2
```

As expected, both M and N can be recovered from M2.

```
d1 = simplify(M2(:,:,:,1)-M);
d2 = simplify(M2(:,:,:,2)-N);
[max(abs(d1(:))) max(abs(d2(:)))]
ans =
     0     0
```

It is also possible to stack M and N along the 1st array dimension, creating a 1-by-3-by-8-by-1 umat.

```
M3 = stack(1,M,N);
size(M3)
ans =
     1     3     8
arraysize(M3)
ans =
     8     1
```

As expected, both M and N can be recovered from M3.

```
d3 = simplify(M3(:,:,1:4)-M);
d4 = simplify(M3(:,:,5:8)-N);
[max(abs(d3(:))) max(abs(d4(:)))]
ans =
     0     0
```

Create Arrays by Assignment

Arrays can be created by direct assignment. As with other MATLAB classes, there is no need to preallocate the variable first. Simply assign elements – all resizing is performed automatically.

For instance, an equivalent construction to

```
a = ureal('a',4);
b = ureal('b',2);
M = stack(1,[a b 1],[-a -b 4+a],[4 5 6],[a 0 0]);
is
Mequiv(1,1,1) = a;
Mequiv(1,2,1) = b;
Mequiv(1,3,1) = 1;
Mequiv(1,:,4) = [a 0 0];
Mequiv(1,:,2:3) = stack(1,[-a -b 4+a],[4 5 6]);
```

The easiest manner for you to verify that the results are the same is to subtract and simplify,

```
d5 = simplify(M-Mequiv);
max(abs(d5(:)))
ans =
     0
```

Binary Operations with Arrays

Most operations simply cycle through the array dimensions, doing pointwise operations. Assume `A` and `B` are `umat` (or `uss`, or `ufrd`) arrays with identical array dimensions (slot 3 and beyond). The operation `C = fcn(A,B)` is equivalent to looping on `k1, k2, ...`, setting

```
C(:,:,k1,k2,...) = fcn(A(:,:,k1,k2,...),B(:,:,k1,k2,...))
```

The result `C` has the same array dimensions as `A` and `B`. The user is required to manage the extra dimensions (i.e., keep track of what they mean). Methods such as permute, squeeze and reshape are included to facilitate this management.

In general, any binary operation requires that the extra-dimensions are compatible. The `umat`, `uss` and `ufrd` objects allow for slightly more flexible interpretation of this. For illustrative purposes, consider a binary operation involving variables *A* and *B*. Suppose the array dimensions of *A* are $n_1 \times \ldots \times n_{l_A}$ and that the array dimensions of *B* are $m_1 \times \ldots \times m_{l_B}$. By MATLAB convention, the infinite number of singleton (i.e., 1) trailing dimensions are not listed. The compatibility of the extra dimensions is determined by the following rule: If *lA* = *lB*, then pad the shorter dimension list with trailing 1's. Now compare the extra dimensions: In the *k-th* dimension, it must be that one of 3 conditions hold: *nk* = *mk*, or *nk* = 1, or *mk* = 1. In other words, non-singleton dimensions must exactly match (so that the pointwise operation can be executed), and singleton dimensions match with anything, implicitly through a `repmat`.

Create Arrays with usample

An extremely common manner in which to generate an array is to sample (in some of the uncertain elements) an uncertain object. Using the `ureal` objects `a` and `b` from above, create a 2-by-3 `umat`.

```
M = [a b;b*b a/b;1-b 1+a*b]
UMAT: 3 Rows, 2 Columns
  a: real, nominal = 4, variability = [-1  1], 3 occurrences
  b: real, nominal = 2, variability = [-1  1], 6 occurrences
size(M)
ans =
     3     2
```

Sample (at 20 random points within its range) the uncertain real parameter `b` in the matrix `M`. This results in a 3-by-2-by-20 `umat`, with only one uncertain element, `a` The uncertain element `b` of `M` has been "sampled out", leaving a new array dimension in its place.

```
[Ms,bvalues] = usample(M,'b',20);
Ms
UMAT: 3 Rows, 2 Columns [array, 20 x 1]
  a: real, nominal = 4, variability = [-1  1], 2 occurrences
size(Ms)
ans =
     3     2    20
```

Continue sampling (at 15 random points within its range) the uncertain real parameter `a` in the matrix `Ms`. This results in a 3-by-2-by-20-by-15 `double`.

```
[Mss,avalues] = usample(Ms,'a',15);
size(Mss)
ans =
     3     2    20    15
class(Mss)
ans =
double
```

The above 2-step sequence can be performed in 1 step,

```
[Mss,values] = usample(M,'b',20,'a',15);
class(Mss)
ans =
double
```

In this case, `values` is a 20-by-15 struct array, with 2 fields `b` and `a`, whose values are the values used in the random sampling. It follows that `usubs(M,values)` is the same as `Mss`.

Rather than sampling the each variable (`a` and `b`) independently, generating a 20-by-15 grid in a 2-dimensional space, the two-dimensional space can be sampled. Sample the 2-dimensional space with 800 points,

```
[Ms,values] = usample(M,{'a' 'b'},800);
size(Ms)
ans =
     3     2   800
size(values)
ans =
   800     1
```

Create Arrays with usubs

Suppose `Values` is a `struct` array, with the following properties: the dimensions of `Value` match the array dimensions of `M`, as described in "Create Arrays with usample" on page 1-63; the field names of `Values` are some (or all) of the names of the uncertain elements of M; and the dimensions of the contents of the fields within `Values` match the sizes of the uncertain elements within `M`. Then `usubs(M,Values)` will substitute the uncertain elements in `M` with the contents found in the respective fields of `Values`.

You can create a 3-by-2 uncertain matrix using two uncertain real parameters.

```
a = ureal('a',4);
b = ureal('b',2);
M = [a b;b*b a/b;1-b 1+a*b];
```

Create a 5-by-1 `struct` array with field name `a`. Make its values random scalars. Create a 1-by-4 `struct` array with field name `b`.

```
Avalue = struct('a',num2cell(rand(5,1)));
Bvalue = struct('b',num2cell(rand(1,4)));
```

Substitute the uncertain real parameter `a` in `M` with the values in `Avalue`, yielding `ma`. Similarly substitute the uncertain real parameter `b` in `M` with the values in `Avalue`, yielding mb.

```
ma = usubs(M,Avalue)
UMAT: 3 Rows, 2 Columns [array, 5 x 1]
  b: real, nominal = 2, variability = [-1  1], 6 occurrences
mb = usubs(M,Bvalue)
UMAT: 3 Rows, 2 Columns [array, 1 x 4]
  a: real, nominal = 4, variability = [-1  1], 2 occurrences
```

Continue, substituting the uncertain real parameter `b` in `ma` with the values in `Bvalue`, yielding `mab`. Do the analogous operation for `mb`, yielding `mba`. Subtract, and note that the difference is 0, as expected.

```
mab = usubs(ma,Bvalue);
mba = usubs(mb,Avalue);
thediff = mab-mba;
max(abs(thediff(:)))
ans =
  4.4409e-016
```

Create Arrays with gridureal

The command `gridureal` enables uniform sampling of specified uncertain real parameters within an uncertain object. It is a specialized case of `usubs`.

`gridureal` removes a specified uncertain real parameter and adds an array dimension (to the end of the existing array dimensions). The new array dimension represents the uniform samples of the uncertain object in the specified uncertain real parameter range.

Create a 2-by-2 uncertain matrix with three uncertain real parameters.

```
a = ureal('a',3,'Range',[2.5 4]);
b = ureal('b',4,'Percentage',15);
c = ureal('c',-2,'Plusminus',[-1 .3]);
M = [a b;b c]
UMAT: 2 Rows, 2 Columns
  a: real, nominal = 3, range = [2.5  4], 1 occurrence
  b: real, nominal = 4, variability = [-15  15]%, 2 occurrences
  c: real, nominal = -2, variability = [-1  0.3], 1 occurrence
```

Grid the uncertain real parameter `b` in `M` with 100 points. The result is a `umat` array, with dependence on uncertain real parameters `a` and `c`.

```
Mgrid1 = gridureal(M,'b',100)
UMAT: 2 Rows, 2 Columns [array, 100 x 1]
  a: real, nominal = 3, range = [2.5  4], 1 occurrence
  c: real, nominal = -2, variability = [-1  0.3], 1 occurrence
```

Operating on the uncertain matrix `M`, grid the uncertain real parameter `a` with 20 points, the uncertain real parameter `b` with 12 points, and the uncertain real parameter `c` with 7 points, The result is a 2-by-2-by20-by-12-by7 `double` array.

```
Mgrid3 = gridureal(M,'a',20,'b',12,'c',7);
size(Mgrid3)
ans =
     2     2    20    12     7
```

Create Arrays with repmat

The MATLAB command `repmat` is used to replicate and tile arrays. It works on the built-in objects of MATLAB, namely `double`, `char`, as well as the generalized container objects `cell` and `struct`. The identical functionality is provided for replicating and tiling uncertain elements (`ureal`, `ultidyn`, etc.) and `umat` objects.

You can create an uncertain real parameter, and replicate it in a 2-by-3 uncertain matrix. Compare to generating the same uncertain matrix through multiplication.

```
a = ureal('a',5);
Amat = repmat(a,[2 3])
UMAT: 2 Rows, 3 Columns
  a: real, nominal = 5, variability = [-1  1], 1 occurrence
Amat2 = a*ones(2,3);
simplify(Amat-Amat2)
ans =
     0     0     0
     0     0     0
```

Create a [4-by-1] `umat` array by stacking four 1-by-3 `umat` objects with the stack command. Use `repmat` to tile this 1-by-3-by-4-by-1 `umat`, into a 2-by-3-by-8-by-5 `umat`.

```
a = ureal('a',4);
b = ureal('b',2);
M = stack(1,[a b 1],[-a -b 4+a],[4 5 6],[a 0 0]);
size(M)
ans =
     1     3     4
Mtiled = repmat(M,[2 1 2 5])
UMAT: 2 Rows, 3 Columns [array, 8 x 5]
  a: real, nominal = 4, variability = [-1  1], 1 occurrence
  b: real, nominal = 2, variability = [-1  1], 1 occurrence
Verify the equality of M and a few certain tiles of Mtiled.
d1 = simplify(M-Mtiled(2,:,5:8,3));
d2 = simplify(M-Mtiled(1,:,1:4,2));
d3 = simplify(M-Mtiled(2,:,1:4,5));
[max(abs(d1(:))) max(abs(d2(:))) max(abs(d3(:)))]
ans =
     0     0     0
```

Note that `repmat` never increases the complexity of the representation of an uncertain object. The number of occurrences of each uncertain element remains the same, regardless of the extent of the replication and tiling.

Create Arrays with repsys

Replicating and tiling uncertain state-space systems (`uss`, and uncertain frequency response data (`ufrd`) is done with `repsys`. The syntax and behavior are the same as the manner in which repmat is used to replicate and tile matrices. The syntax and behavior of `repsys` for `uss` and `ufrd` objects are the same as the traditional `repsys` which operates on `ss` and `frd` objects. Just as in those cases, the uncertain version of `repsys` also allows for diagonal tiling.

Using permute and ipermute

The commands `permute` and `ipermute` are generalizations of `transpose`, which exchanges the rows and columns of a two-dimensional matrix.

`permute(A,ORDER)` rearranges the dimensions of `A` so that they are in the order specified by the vector `ORDER`. The array produced has the same values of `A` but the order of the subscripts needed to access any particular element are rearranged as specified by `ORDER`. The elements of `ORDER` must be a rearrangement of the numbers from 1 to `N`.

All of the uncertain objects are essentially 2-dimensional (output and input) operators with array dependence. This means that the first 2 dimensions are treated differently from dimensions 3 and beyond. It is not permissible to permute across these groups.

For `uss` and `ufrd`, the restriction is built into the syntax. The elements of the `ORDER` vector only refer to array dimensions. Therefore, there is no possibility of permute across these dimensions. In you need to permute the first two dimensions, use the command `transpose` instead.

For `umat`, the restriction is enforced in the software. The elements of the `ORDER` vector refer to all dimensions. However, the first two elements of `ORDER` must be a rearrangement of the numbers 1 and 2. The remaining elements of `ORDER` must be a rearrangement of the numbers 3 through N. If either of those conditions fail, an error is generated. Hence, for `umat` arrays, either `permute` or `transpose` can be used to effect the transpose operation.

Decomposing Uncertain Objects

Each uncertain object (`umat`, `uss`, `ufrd`) is a generalized feedback connection (`lft`) of a not-uncertain object (e.g., `double`, `ss`, `frd`) with a diagonal augmentation of uncertain elements (`ureal`, `ultidyn`, `ucomplex`, `ucomplexm`, `udyn`). In robust control jargon, if the uncertain elements are normalized, this decomposition is often called "the M/D form."

The purpose of the uncertain objects (`ureal`, `ultidyn`, `umat`, `uss`, etc.) is to hide this underlying decomposition, and allow the user to focus on modeling and analyzing uncertain systems, rather than the details of correctly propagating the M/D representation in manipulations. Nevertheless, advanced users may want access to the familiar M/D form. The command `lftdata` accomplishes this decomposition.

Since `ureal`, `ucomplex` and `ucomplexm` do not have their `NominalValue` necessarily at zero, and in the case of `ureal` objects, are not symmetric about the `NominalValue`, some details are required in describing the decomposition.

Normalizing Functions for Uncertain Elements

Associated with each uncertain element is a normalizing function. The normalizing function maps the uncertain element into a normalized uncertain element.

If ρ is an uncertain real parameter, with range `[L R]` and nominal value `N`, then the normalizing function F is

$$F(\rho) = \frac{A + B\rho}{C + D\rho}$$

with the property that for all ρ satisfying $L \le \rho \le R$, it follows that $-1 \le F(\rho) \le 1$, moreover, $F(L) = -1$, $F(N) = 0$, and $F(R) = 1$. If the nominal value is centered in the range, then it is easy to conclude that

$$A = \frac{R + L}{R - L}$$
$$B = \frac{2}{R - L}$$
$$C = 1$$
$$D = 0.$$

It is left as an algebra exercise for the user to work out the various values for `A, B, C` and `D` when the nominal value is not centered.

If E is an uncertain gain-bounded, linear, time-invariant dynamic uncertainty, with gain-bound ß, then the normalizing function F is

$$F(E) = \frac{1}{\beta}E.$$

If E is an uncertain positive-real, linear, time-invariant dynamic uncertainty, with positivity bound ß, then the normalizing function F is

$$F(E) = \left[I - \alpha\left(E - \frac{\beta}{2}I\right)\right]\left[I + \alpha\left(E - \frac{\beta}{2}I\right)\right]^{-1}$$

where $\alpha = 2|\beta| + 1$.

The normalizing function for an uncertain complex parameter ξ, with nominal value C and radius γ, is

$$F(\xi) = \frac{1}{\gamma}(\xi - C).$$

The normalizing function for uncertain complex matrices H, with nominal value N and weights W_L and W_R is

$$F(H) = W_L^{-1}(H - N)W_R^{-1}$$

In each case, as the uncertain element varies over its range, the absolute value of the normalizing function (or norm, in the matrix case) varies from 0 and 1.

Properties of the Decomposition

Take an uncertain object A, dependent on uncertain real parameters $\rho_1,...,\rho_N$, uncertain complex parameters $\xi_1,...,\xi_K$, uncertain complex matrices $H_1,...,H_B$, uncertain gain-bounded linear, time-invariant dynamics $E_1,...,E_D$, and uncertain positive-real linear, time-invariant dynamics $P_1,...,P_Q$.

Write $A(\rho,\xi,H,E,P)$ to indicate this dependence. Using `lftdata`, A can be decomposed into two separate pieces: M and $\Delta(\rho,\xi,H,E,P)$ with the following properties:

- M is certain (i.e., if A is `uss`, then M is `ss`; if A is `umat`, then M is `double`; if A is `ufrd`, then M is `frd`).
- Δ is always a `umat`, depending on the same uncertain elements as A, with ranges, bounds, weights, etc., unaltered.
- The form of Δ is block diagonal, with elements made up of the normalizing functions acting on the individual uncertain elements:

$$\Delta(\rho,\xi,H,E,P)=\begin{bmatrix} F(\rho) & 0 & 0 & 0 & 0 \\ 0 & F(\xi) & 0 & 0 & 0 \\ 0 & 0 & F(H) & 0 & 0 \\ 0 & 0 & 0 & F(E) & 0 \\ 0 & 0 & 0 & 0 & F(P) \end{bmatrix}.$$

- $A(\rho,\xi,H,E,P)$ is given by a linear fractional transformation of M and $\Delta(\rho,\xi,H,E,P)$,

$$A(\rho,\xi)=M_{22}+M_{21}\Delta(\rho,\xi,H,E,P)\left[I-M_{11}\Delta(\rho,\xi,H,E,P)\right]^{-1}M_{12}.$$

The order of the normalized elements making up `A` is not the simple order shown above. It is actually the same order as given by the command `fieldnames(M.Uncertainty)`. See “Advanced Syntax of lftdata” on page 1-75 for more information.

Decompose Uncertain Model Using lftdata

You decompose an uncertain model into a fixed certain part and normalized uncertain part using the lftdata command. To see how this command works, create a 2-by-2 uncertain matrix (`umat`) using three uncertain real parameters.

```
delta = ureal('delta',2);
eta = ureal('eta',6);
rho = ureal('rho',-1);
A = [3+delta+eta delta/eta;7+rho rho+delta*eta]
```

```
A =
```

```
  Uncertain matrix with 2 rows and 2 columns.
  The uncertainty consists of the following blocks:
    delta: Uncertain real, nominal = 2, variability = [-1,1], 2 occurrences
    eta: Uncertain real, nominal = 6, variability = [-1,1], 3 occurrences
    rho: Uncertain real, nominal = -1, variability = [-1,1], 1 occurrences

Type "A.NominalValue" to see the nominal value, "get(A)" to see all properties, and "A.
```

The `umat` `A` depends on two occurrences of `delta`, three occurrences of `eta`, and one occurrence of `rho`.

Decompose `A` into `M` and `Delta`.

```
[M,Delta] = lftdata(A);
```

`M` is a numeric matrix.

```
M
```

```
M =

  Columns 1 through 7

         0         0         0   -0.1667         0         0    1.0000
         0         0         0         0    1.0000         0         0
         0         0         0         0         0         0    1.0000
         0         0         0   -0.1667         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0    1.0000
    1.0000         0    1.0000   -0.3333         0         0   11.0000
         0    1.0000         0         0    2.0000    1.0000    6.0000

  Column 8

    0.1667
    6.0000
         0
    0.1667
    1.0000
    1.0000
    0.3333
   11.0000
```

`Delta` is a `umat` with the same uncertainty dependence as `A`.

```
Delta
```

```
Delta =

  Uncertain matrix with 6 rows and 6 columns.
  The uncertainty consists of the following blocks:
    delta: Uncertain real, nominal = 2, variability = [-1,1], 2 occurrences
    eta: Uncertain real, nominal = 6, variability = [-1,1], 3 occurrences
    rho: Uncertain real, nominal = -1, variability = [-1,1], 1 occurrences

Type "Delta.NominalValue" to see the nominal value, "get(Delta)" to see all properties
```

To examine some of the characteristics of `Delta`, sample it at three points. Note that:

- The sampled value of `Delta` is always diagonal.
- The sampled values alway range between -1 and 1, because `Delta` is normalized.
- The sampled matrices each contain three independent values. Duplication of the entries is consistent with the dependence of `Delta` and `A` on the three uncertain real parameters.

```
usample(Delta,3)
```

```
ans(:,:,1) =

    0.6294         0         0         0         0         0
         0    0.6294         0         0         0         0
         0         0    0.8268         0         0         0
         0         0         0    0.8268         0         0
         0         0         0         0    0.8268         0
         0         0         0         0         0   -0.4430


ans(:,:,2) =

    0.8116         0         0         0         0         0
         0    0.8116         0         0         0         0
         0         0    0.2647         0         0         0
         0         0         0    0.2647         0         0
         0         0         0         0    0.2647         0
```

```
         0         0         0         0         0    0.0938


ans(:,:,3) =

   -0.7460         0         0         0         0         0
         0   -0.7460         0         0         0         0
         0         0   -0.8049         0         0         0
         0         0         0   -0.8049         0         0
         0         0         0         0   -0.8049         0
         0         0         0         0         0    0.9150
```

Verify that the maximum gain of `Delta` is 1.

```
maxnorm = wcnorm(Delta)
```

```
maxnorm =

  struct with fields:

    LowerBound: 0
    UpperBound: 1.0008
```

Finally, verify that `lft(Delta,M)` is the same as `A`. To do so, take the difference, and use the `'full'` option in `simplify` to remove redundant dependencies on uncertain elements.

```
simplify(lft(Delta,M)-A,'full')
```

```
ans =

     0     0
     0     0
```

Advanced Syntax of lftdata

Even for the advanced user, the variable `Delta` will actually not be that useful, as it is still a complex object. On the other hand, its internal structure is described completely using a 3rd (and 4th) output argument.

```
[M,Delta,BlkStruct,NormUnc] = lftdata(A);
```

The rows of `BlkStruct` correspond to the uncertain elements named in `fieldnames(A.Uncertainty)`. Note that the range/bound information about each uncertain element is not included in `BlkStruct`.

The elements of `BlkStruct` describe the size, type and number-of-copies of the uncertain elements in `A`, and implicitly delineate the exact block-diagonal structure of `Delta`. Note that the range/bound information about each uncertain element is not included in `BlkStruct`.

```
BlkStruct(1)
ans =
           Name: 'delta'
           Size: [1 1]
           Type: 'ureal'
    Occurrences: 2
BlkStruct(2)
ans =
           Name: 'eta'
           Size: [1 1]
           Type: 'ureal'
    Occurrences: 3
BlkStruct(3)
ans =
           Name: 'rho'
           Size: [1 1]
           Type: 'ureal'
    Occurrences: 1
```

Together, these mean `Delta` is a block diagonal augmentation of the normalized version of 3 uncertain elements.

The first element is named `'delta'`. It is 1-by-1; it is of class `ureal`; and there are 2 copies diagonally augmented.

The second element is named `'eta'`. It is 1-by-1; it is of class `ureal`; and there are 3 copies diagonally augmented.

The third element is named `'rho'`. It is 1-by-1; it is of class `ureal`; and there is 1 copy,

The 4th output argument contains a cell array of normalized uncertain elements. The cell array contains as many occurrences of each element as there are occurrences in the original uncertain object A.

```
size(NormUnc)
ans =
     6     1
NormUnc{1}
Uncertain Real Parameter: Name deltaNormalized, NominalValue 0,
variability = [-1  1]
isequal(NormUnc{2},NormUnc{1})
ans =
     1
NormUnc{3}
Uncertain Real Parameter: Name etaNormalized, NominalValue 0,
variability = [-1  1]
isequal(NormUnc{4},NormUnc{3})
ans =
     1
isequal(NormUnc{5},NormUnc{3})
ans =
     1
NormUnc{6}
Uncertain Real Parameter: Name rhoNormalized, NominalValue 0,
variability = [-1  1]
```

Each normalized element has `'Normalized'` appended to its original name to avoid confusion. When normalized,

- `ureal` objects have nominal value of 0, and range from -1 to 1.
- `ultidyn` objects are norm bounded, with norm bound of 1.
- `ucomplex` objects have nominal value of 0, and radius 1.
- `ucomplexm` objects have nominal value of 0, and identity matrices for each of the `WL` and`WR` weights.

The possible behaviors of `Delta` and `blkdiag(NormUnc{:})` are the same. Consequently, the possible behaviors of `A` and `lft(blkdiag(NormUnc{:}),M)` are the same.

Hence, by manipulating `M, BlkStruct` and `NormUnc`, a power-user has direct access to all of the linear fractional transformation details, and can easily work at the level of the theorems and algorithms that underlie the methods.

See Also

`lftdata`

Related Examples

- “Introduction to Uncertain Elements” on page 1-2

2

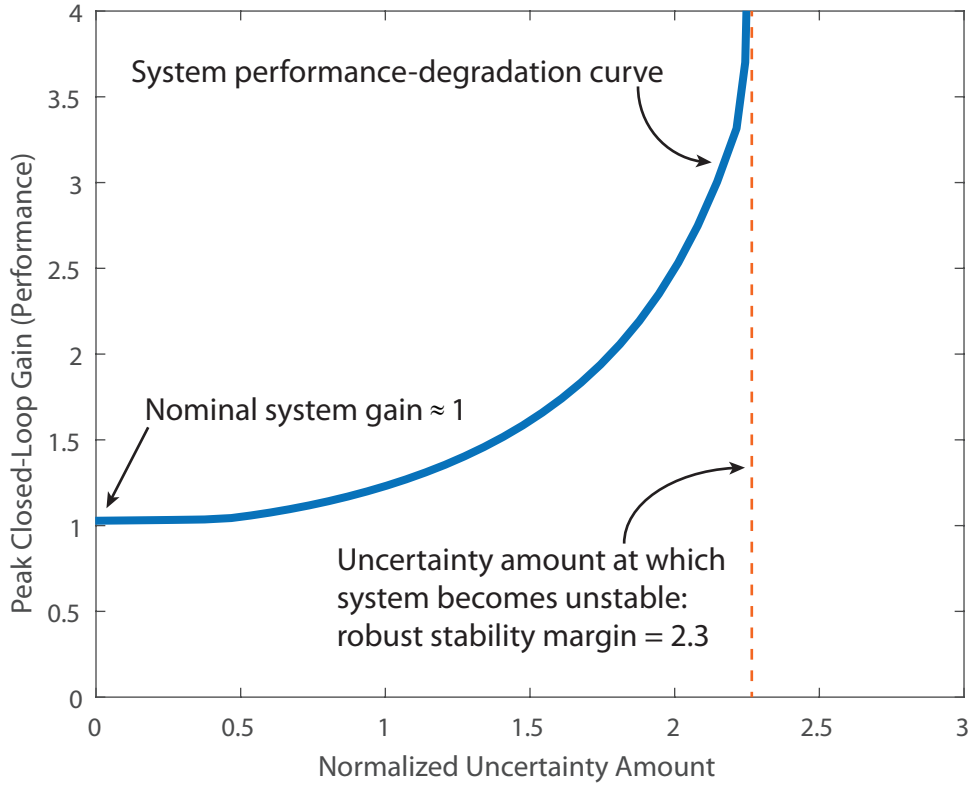
Generalized Robustness Analysis

Robustness and Worst-Case Analysis

In robust control design, performance is expressed and measured in terms of the peak gain (the H_∞ norm or peak singular value) of a system. The smaller this gain is, the better the system performance. The performance of a nominally stable uncertain system generally degrades as the amount of uncertainty increases. Use robustness analysis and worst-case analysis to examine how the amount of uncertainty in your system affects the stability and peak gain of the system.

Robustness Analysis

Robustness analysis is about finding the maximum amount of uncertainty compatible with stability or with a given performance level. The following illustration shows a typical trade-off curve between performance and robustness. Here, the peak gain (peak magnitude on a Bode plot or singular-value plot) characterizes the system performance.



The x-axis quantifies the normalized amount of uncertainty. The value $x = 1$ corresponds to the uncertainty ranges specified in the model. $x = 2$ represents the system with twice as much uncertainty. $x = 0$ corresponds to the nominal system. (See `actual2normalized` for more details about normalized uncertainty ranges.) The y-axis is performance, measured as the peak gain of some closed-loop transfer function. For instance, if the closed-loop transfer function measures the sensitivity of an error signal to some disturbance, then higher peak gain corresponds to poorer disturbance rejection.

When all uncertain elements are set to their nominal values ($x = 0$), the gain of the system is its nominal value. In the figure, the nominal system gain is about 1. As the range of values that the uncertain elements can take increases, the peak gain over the uncertainty range increases. The heavy blue line represents the peak gain, and is called

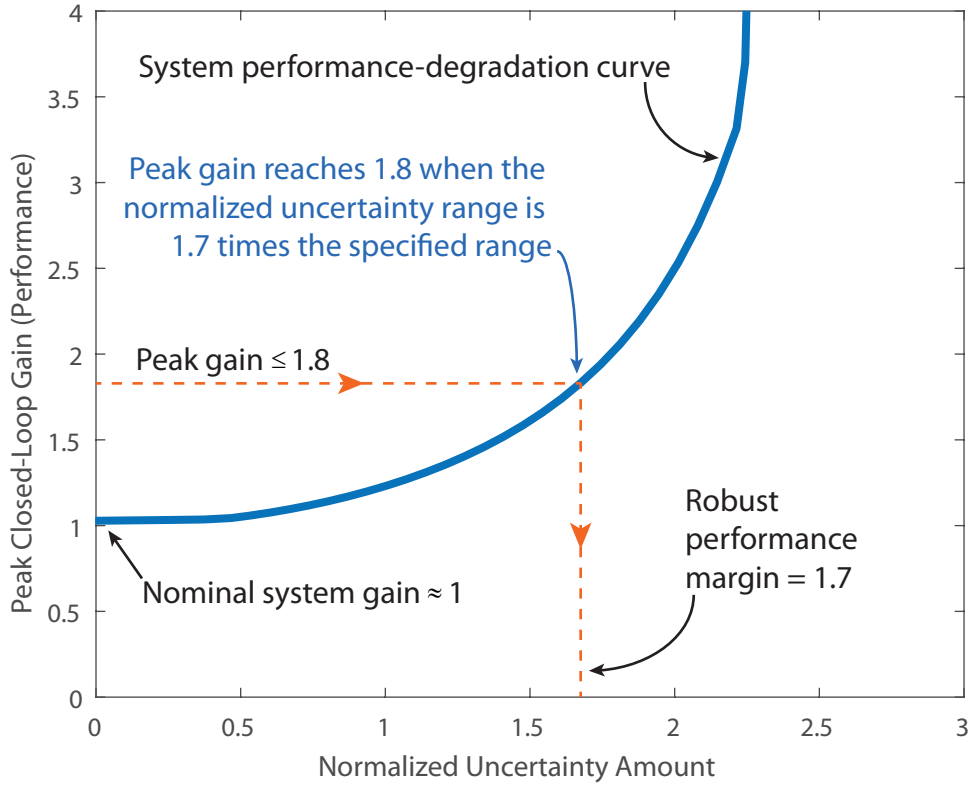
the *system performance degradation curve*. It increases monotonically as a function of the uncertainty amount.

Robust Stability Margin

The system performance degradation curve typically has a vertical asymptote corresponding to the *robust stability margin*. This margin is the maximum amount of uncertainty that the system can tolerate while remaining stable. For the system of the previous illustration, the peak gain becomes infinite at around $x = 2.3$. In other words, the system becomes unstable when the uncertainty range is 2.3 times that specified in the model (in normalized units). Therefore, the robust stability margin is 2.3. To compute the robust stability margin for an uncertain system model, use the `robstab` function.

Robust Performance Margin

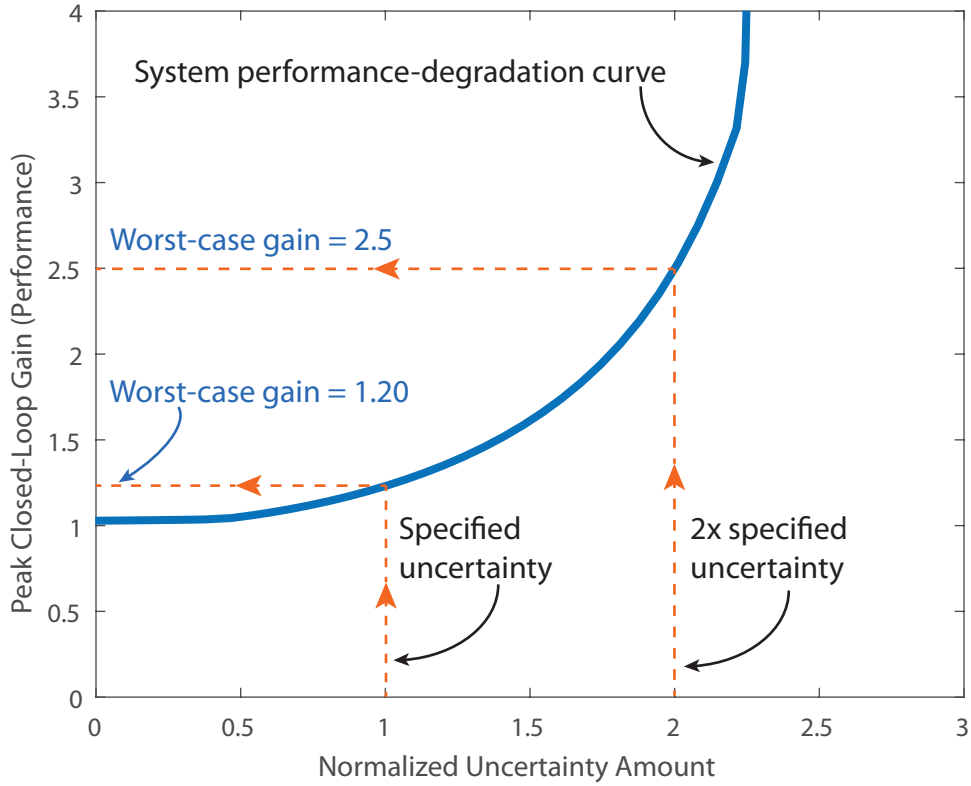
The *robust performance margin* for a given gain, γ, is the maximum amount of uncertainty the system can tolerate while having a peak gain less than γ. For example, in the following illustration, suppose that you want to keep the peak closed-loop gain below 1.8. For that peak gain, the robust performance margin is about 1.7. This value means that the peak gain of the system remains below 1.8 as long as the uncertainty remains within 1.7 times the specified uncertainty (in normalized units).



To compute the robust performance margin for an uncertain system model, use the `robgain` function.

Worst-Case Gain Measure

The *worst-case gain* is the largest value that the peak gain can take over a specific uncertainty range. This value is the counterpart of the robust performance margin. While the robust performance margin measures the maximum amount of uncertainty compatible with a particular peak gain level, the worst-case gain measures the maximum gain associated with a particular uncertainty amount. For instance, in the following illustration, the worst-case gain for the uncertainty amount specified in the model is about 1.20. If that uncertainty amount is doubled, the worst-case gain increases to 2.5.



To compute the worst-case gain for an uncertain system model, use the `wcgain` function. The `ULevel` option of the `wcOptions` command lets you compute the worst-case gain for different amounts of uncertainty.

See Also

`robgain` | `robstab` | `wcgain`

Related Examples

- "Robust Stability and Worst-Case Gain of Uncertain System"
- "MIMO Robustness Analysis"

Robust Stability, Robust Performance and Mu Analysis

This example shows how to use Robust Control Toolbox™ to analyze and quantify the robustness of feedback control systems. It also provides insight into the connection with mu analysis and the `mussv` function.

System Description

Figure 1 shows the block diagram of a closed-loop system. The plant model P is uncertain and the plant output y must be regulated to remain small in the presence of disturbances d and measurement noise n.

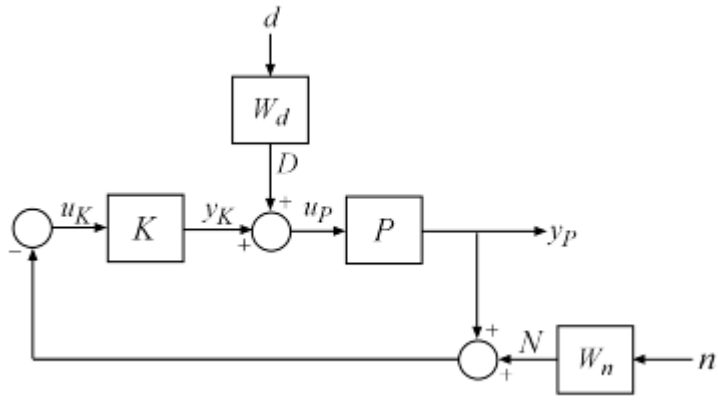


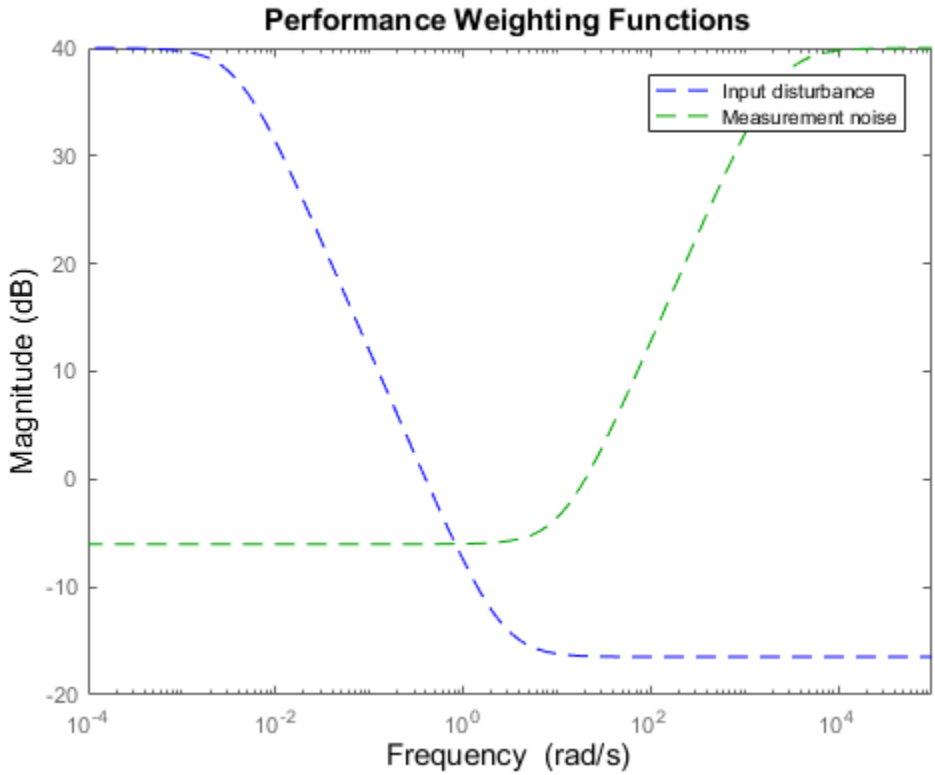
Figure 1: Closed-loop system for robustness analysis

Disturbance rejection and noise insensitivity are quantified by the performance objective

$$\left\| (P(1+KP)^{-1}W_d, (1+PK)^{-1}W_n) \right\|_\infty$$

where W_d and W_n are weighting functions reflecting the frequency content of d and n. Here W_d is large at low frequencies and W_n is large at high frequencies.

```
Wd = makeweight(100,.4,.15);
Wn = makeweight(0.5,20,100);
bodemag(Wd,'b--',Wn,'g--')
title('Performance Weighting Functions')
legend('Input disturbance','Measurement noise')
```



Creating an Uncertain Plant Model

The uncertain plant model P is a lightly-damped, second-order system with parametric uncertainty in the denominator coefficients and significant frequency-dependent unmodeled dynamics beyond 6 rad/s. The mathematical model looks like:

$$P(s) = \frac{16}{s^2 + 0.16s + k}(1 + W_u(s)\delta(s))$$

The parameter k is assumed to be about 40% uncertain, with a nominal value of 16. The frequency-dependent uncertainty at the plant input is assumed to be about 30% at low frequency, rising to 100% at 10 rad/s, and larger beyond that. Construct the uncertain plant model P by creating and combining the uncertain elements:

```
k = ureal('k',16,'Percentage',30);
delta = ultidyn('delta',[1 1],'SampleStateDim',4);
Wu = makeweight(0.3,10,20);
P = tf(16,[1 0.16 k]) * (1+Wu*delta);
```

Designing a Controller

We use the controller designed in the example "Improving Stability While Preserving Open-Loop Characteristics". The plant model used there happens to be the nominal value of the uncertain plant model created above. For completeness, we repeat the commands used to generate the controller.

```
K_PI = pid(1,0.8);
K_rolloff = tf(1,[1/20 1]);
Kprop = K_PI*K_rolloff;
[negK,~,Gamma] = ncfsyn(P.NominalValue,-Kprop);
K = -negK;
```

Closing the Loop

Use `connect` to build an uncertain model of the closed-loop system of Figure 1. Name the signals coming in and out of each block and let `connect` do the wiring:

```
P.u = 'uP';  P.y = 'yP';
K.u = 'uK';  K.y = 'yK';
S1 = sumblk('uP = yK + D');
S2 = sumblk('uK = -yP - N');
Wn.u = 'n'; Wn.y = 'N';
Wd.u = 'd'; Wd.y = 'D';
ClosedLoop = connect(P,K,S1,S2,Wn,Wd,{'d','n'},'yP');
```

The variable `ClosedLoop` is an uncertain system with two inputs and one output. It depends on two uncertain elements: a real parameter `k` and an uncertain linear, time-invariant dynamic element `delta`.

```
ClosedLoop
```

```
ClosedLoop =

  Uncertain continuous-time state-space model with 1 outputs, 2 inputs, 10 states.
  The model uncertainty consists of the following blocks:
    delta: Uncertain 1x1 LTI, peak gain = 1, 1 occurrences
    k: Uncertain real, nominal = 16, variability = [-30,30]%, 1 occurrences
```

```
Type "ClosedLoop.NominalValue" to see the nominal value, "get(ClosedLoop)" to see all p
```

Robust Stability Analysis

The classical margins from `allmargin` indicate good stability robustness to unstructured gain/phase variations within the loop.

```
allmargin(P.NominalValue*K)
```

```
ans =

  struct with fields:

     GainMargin: [2.1692e-17 6.3299 11.1423]
    GMFrequency: [0 1.6110 15.1667]
    PhaseMargin: [80.0276 -99.6641 63.7989]
    PMFrequency: [0.4472 3.1460 5.2319]
    DelayMargin: [3.1236 1.4443 0.2128]
    DMFrequency: [0.4472 3.1460 5.2319]
         Stable: 1
```

Does the closed-loop system remain stable for all values of `k`, `delta` in the ranges specified above? Answering this question requires a more sophisticated analysis using the `robstab` function.

```
[stabmarg,wcu] = robstab(ClosedLoop);
stabmarg
```

```
stabmarg =

  struct with fields:

          LowerBound: 1.4673
          UpperBound: 1.4702
    CriticalFrequency: 5.8651
```

The variable `stabmarg` gives upper and lower bounds on the **robust stability margin**, a measure of how much uncertainty on `k`, `delta` the feedback loop can tolerate before becoming unstable. For example, a margin of 0.8 indicates that as little as 80% of the

specified uncertainty level can lead to instability. Here the margin is about 1.5, which means that the closed loop will remain stable for up to 150% of the specified uncertainty.

The variable `wcu` contains the combination of `k` and `delta` closest to their nominal values that causes instability.

```
wcu
```

```
wcu =

  struct with fields:

    delta: [1×1 ss]
        k: 23.0571
```

We can substitute these values into `ClosedLoop` and verify that these values cause the closed-loop system to be unstable.

```
format short e
pole(usubs(ClosedLoop,wcu))
```

```
ans =

  -2.0932e+02 + 0.0000e+00i
  -1.3468e+02 + 0.0000e+00i
  -1.4890e+01 + 6.1489e+00i
  -1.4890e+01 - 6.1489e+00i
  -9.3519e-01 + 0.0000e+00i
   2.0921e-14 + 5.8651e+00i
   2.0921e-14 - 5.8651e+00i
  -3.1259e+00 + 1.8846e+00i
  -3.1259e+00 - 1.8846e+00i
  -3.9097e-01 + 0.0000e+00i
  -1.9998e+01 + 0.0000e+00i
  -2.3093e+03 + 0.0000e+00i
  -3.9549e-03 + 0.0000e+00i
```

Note that the natural frequency of the unstable closed-loop pole is given by `stabmarg.CriticalFrequency`:

```
stabmarg.CriticalFrequency
```

```
ans =

   5.8651e+00
```

Connection with Mu Analysis

The structured singular value, or μ, is the mathematical tool used by `robstab` to compute the robust stability margin. If you are comfortable with structured singular value analysis, you can use the `mussv` function directly to compute mu as a function of frequency and reproduce the results above. The function `mussv` is the underlying engine for all robustness analysis commands.

To use `mussv`, we first extract the `(M,Delta)` decomposition of the uncertain closed-loop model `ClosedLoop`, where `Delta` is a block-diagonal matrix of (normalized) uncertain elements. The 3rd output argument of `lftdata`, `BlkStruct`, describes the block-diagonal structure of `Delta` and can be used directly by `mussv`

```
[M,Delta,BlkStruct] = lftdata(ClosedLoop);
```

For robust stability analysis, only the channels of `M` associated with the uncertainty channels are used. Based on the row/column size of `Delta`, select the proper columns and rows of `M`. Remember that the rows of `Delta` correspond to the columns of `M`, and vice versa. Consequently, the column dimension of `Delta` is used to specify the rows of `M`:

```
szDelta = size(Delta);
M11 = M(1:szDelta(2),1:szDelta(1));
```

In its simplest form, mu-analysis is performed on a finite grid of frequencies. Pick a vector of logarithmically-spaced frequency points and evaluate the frequency response of `M11` over this frequency grid.

```
omega = logspace(-1,2,50);
M11_g = frd(M11,omega);
```

Compute `mu(M11)` at these frequencies and plot the resulting lower and upper bounds:

```
mubnds = mussv(M11_g,BlkStruct,'s');

LinMagopt = bodeoptions;
LinMagopt.PhaseVisible = 'off'; LinMagopt.XLim = [1e-1 1e2]; LinMagopt.MagUnits = 'abs
bodeplot(mubnds(1,1),mubnds(1,2),LinMagopt);
xlabel('Frequency (rad/sec)');
```

```
ylabel('Mu upper/lower bounds');
title('Mu plot of robust stability margins (inverted scale)');
```

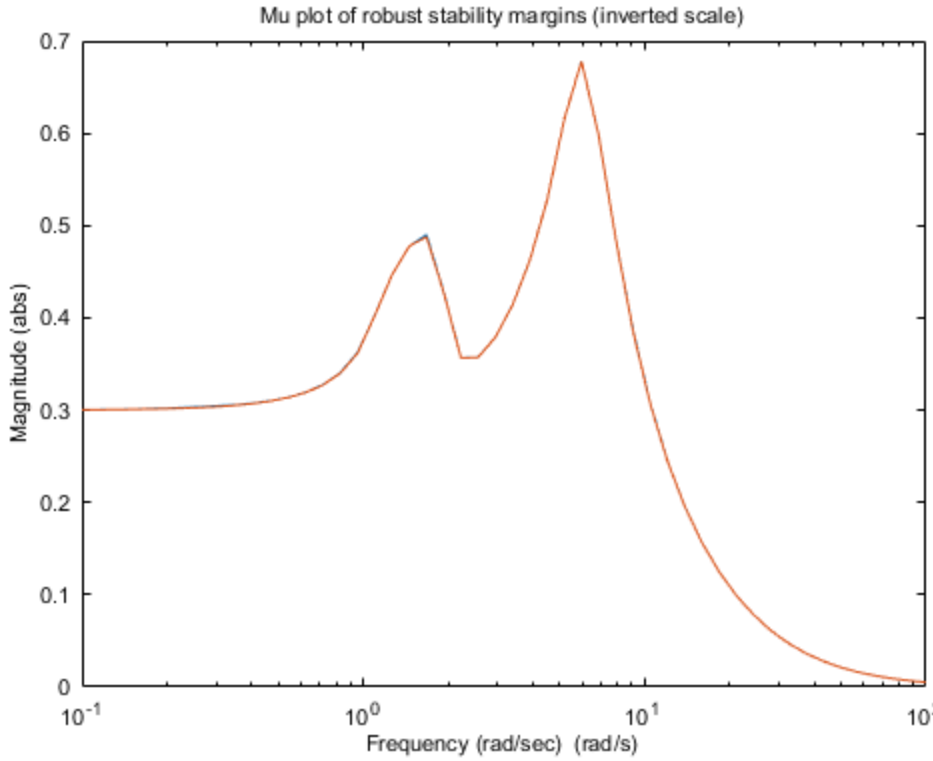


Figure 3: Mu plot of robust stability margins (inverted scale)

The robust stability margin is the reciprocal of the structured singular value. Therefore upper bounds from `mussv` become lower bounds on the stability margin. Make these conversions and find the destabilizing frequency where the mu upper bound peaks (that is, where the stability margin is smallest):

```
[pkl,wPeakLow] = getPeakGain(mubnds(1,2));
[pku] = getPeakGain(mubnds(1,1));
SMfromMU.LowerBound = 1/pku;
SMfromMU.UpperBound = 1/pkl;
```

```
SMfromMU.CriticalFrequency = wPeakLow;
```

Compare `SMfromMU` to the bounds `stabmarg` computed with `robstab`. The values are in rough agreement with `robstab` yielding slightly weaker margins. This is because `robstab` uses a more sophisticated approach than frequency gridding and can accurately compute the peak value of `mu` across frequency.

```
stabmarg
SMfromMU
```

```
stabmarg =

  struct with fields:

          LowerBound: 1.4673e+00
          UpperBound: 1.4702e+00
    CriticalFrequency: 5.8651e+00


SMfromMU =

  struct with fields:

          LowerBound: 1.4746e+00
          UpperBound: 1.4748e+00
    CriticalFrequency: 5.9636e+00
```

Robust Performance Analysis

For the nominal values of the uncertain elements `k` and `delta`, the closed-loop gain is less than 1:

```
getPeakGain(ClosedLoop.NominalValue)
```

```
ans =

   9.8040e-01
```

This says that the controller `K` meets the disturbance rejection and noise insensitivity goals. But is this nominal performance maintained in the face of the modeled uncertainty? This question is best answered with `robgain`.

```
opt = robOptions('Display','on');
[perfmarg,wcu] = robgain(ClosedLoop,1,opt);
```

```
Computing peak...  Percent completed: 100/100
The performance level 1 is not robust to the modeled uncertainty.
 -- The gain remains below 1 for up to 39.9% of the modeled uncertainty.
 -- There is a bad perturbation amounting to 39.9% of the modeled uncertainty.
 -- This perturbation causes a gain of 1 at the frequency 0.134 rad/seconds.
```

The answer is negative: `robgain` found a perturbation amounting to only 40% of the specified uncertainty that drives the closed-loop gain to 1.

```
getPeakGain(usubs(ClosedLoop,wcu),1e-6)
```

```
ans =

   1.0000e+00
```

This suggests that the closed-loop gain will exceed 1 for 100% of the specified uncertainty. This is confirmed by computing the worst-case gain:

```
wcg = wcgain(ClosedLoop)
```

```
wcg =

  struct with fields:

          LowerBound: 1.5714e+00
          UpperBound: 1.5749e+00
    CriticalFrequency: 5.9583e+00
```

The worst-case gain is about 1.6. This analysis shows that while the controller `K` meets the disturbance rejection and noise insensitivity goals for the nominal plant, it is unable to maintain this level of performance for the specified level of plant uncertainty.

See Also

`mussv` | `robgain` | `robstab` | `wcgain`

Related Examples

- “Getting Reliable Estimates of Robustness Margins” on page 2-17

More About

- "Robustness and Worst-Case Analysis" on page 2-2

Getting Reliable Estimates of Robustness Margins

This example illustrates the pitfalls of using frequency gridding to compute robustness margins for systems with only real uncertain parameters. It presents a safer approach along with ways to mitigate discontinuities in the structured singular value μ.

How Discontinuities Can Hide Robustness Issues

Consider a spring-mass-damper system with 100% parameter uncertainty in the damping coefficient and 0% uncertainty in the spring coefficient. Note that all uncertainty is of `ureal` type.

```
m = 1;
k = 1;
c = ureal('c',1,'plusminus',1);
sys = tf(1,[m c k]);
```

As the uncertain element `c` varies, the only place where the poles can migrate from stable to unstable is at s = j*1 (1 rad/sec). No amount of variation in `c` can cause them to migrate across the jw-axis at any other frequency. As a result, the robust stability margin is infinite at all frequencies except 1 rad/s, where the margin with respect to the specified uncertainty is 1. In other words, the robust stability margin and the underlying structured singular value μ are discontinuous as a function of frequency.

The traditional approach to computing the robust stability margin is to pick a frequency grid and compute lower and upper bounds for μ at each frequency point. Under most conditions, the robust stability margin is continuous with respect to frequency and this approach gives good estimates provided you use a sufficiently dense frequency grid. However in problems with only `ureal` uncertainty, such as the example above, poles can migrate from stable to unstable only at specific frequencies (the points of discontinuity for μ), so any frequency grid that excludes these particular frequencies will lead to over-optimistic stability margins.

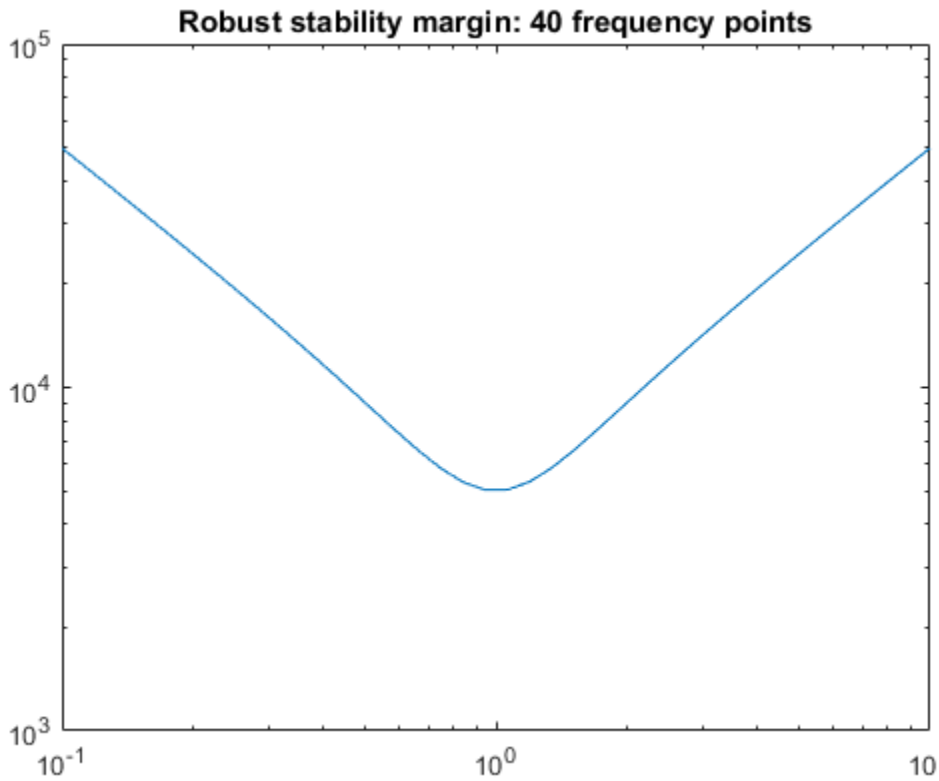
To see this effect, pick a frequency grid for the spring-mass-damper system above and compute the robust stability margins at these frequency points using `robstab`.

```
omega = logspace(-1,1,40); % one possible grid
[stabmarg,wcu,info] = robstab(sys,omega);
stabmarg
```

```
stabmarg =

  struct with fields:

          LowerBound: 5.0348e+03
          UpperBound: Inf
    CriticalFrequency: 0.1000
```

The field `info.Bounds` gives the margin lower and upper bounds at each frequency. Verify that the lower bound (the guaranteed margin) is large at all frequencies.

```
loglog(omega,info.Bounds(:,1))
title('Robust stability margin: 40 frequency points')
```



Note that making the grid denser would not help. Only by adding f=1 to the grid will we find the true margin.

```
f = 1;
stabmarg = robstab(sys,f)
```

```
stabmarg =

  struct with fields:

          LowerBound: 1.0000
          UpperBound: 1
    CriticalFrequency: 1
```

Safe Computation of Robustness Margins

Rather than specifying a frequency grid, apply `robstab` directly to the USS model `sys`. This uses a more advanced algorithm that is guaranteed to find the peak of μ even in the presence of a discontinuity. This approach is more accurate and often faster than frequency gridding.

```
[stabmarg,wcu] = robstab(sys)
```

```
stabmarg =

  struct with fields:

          LowerBound: 1.0000
          UpperBound: 1.0000
    CriticalFrequency: 1.0000


wcu =

  struct with fields:

    c: 2.2204e-16
```

This computes the correct robust stability margin (1), identifies the critical frequency (`f=1`), and finds the smallest destabilizing perturbation (setting `c=0`, as expected).

Modifying the Uncertainty Model to Eliminate Discontinuities

The example above shows that the robust stability margin can be a discontinuous function of frequency. In other words, it can have jumps. We can eliminate such jumps by adding a small amount of uncertain dynamics to every uncertain real parameter. This amounts to adding some dynamics to pure gains. Importantly, as the size of the added dynamics goes to zero, the estimated margin for the modified problem converges to the true margin for the original problem.

In the spring-mass-damper example, we model `c` as a `ureal` with the range [0.05,1.95] rather than [0,2], and add a `ultidyn` perturbation with gain bounded by 0.05. This combination covers the original uncertainty in `c` and introduces only 5% conservatism.

```
cc = ureal('cReal',1,'plusminus',0.95) + ultidyn('cUlti',[1 1],'Bound',0.05);
sysreg = usubs(sys,'c',cc);
```

Recompute the robust stability margin over the frequency grid `omega`.

```
[stabmarg,~,info] = robstab(sysreg,omega);
stabmarg
```

```
stabmarg =

  struct with fields:

          LowerBound: 2.3629
          UpperBound: 2.3630
    CriticalFrequency: 0.9427
```

Now the frequency-gridded calculation yields a margin of 2.36. This is still greater than 1 (the true margin) because the density of frequency points is not high enough. Increase the number of points from 40 to 200 and recompute the margin.

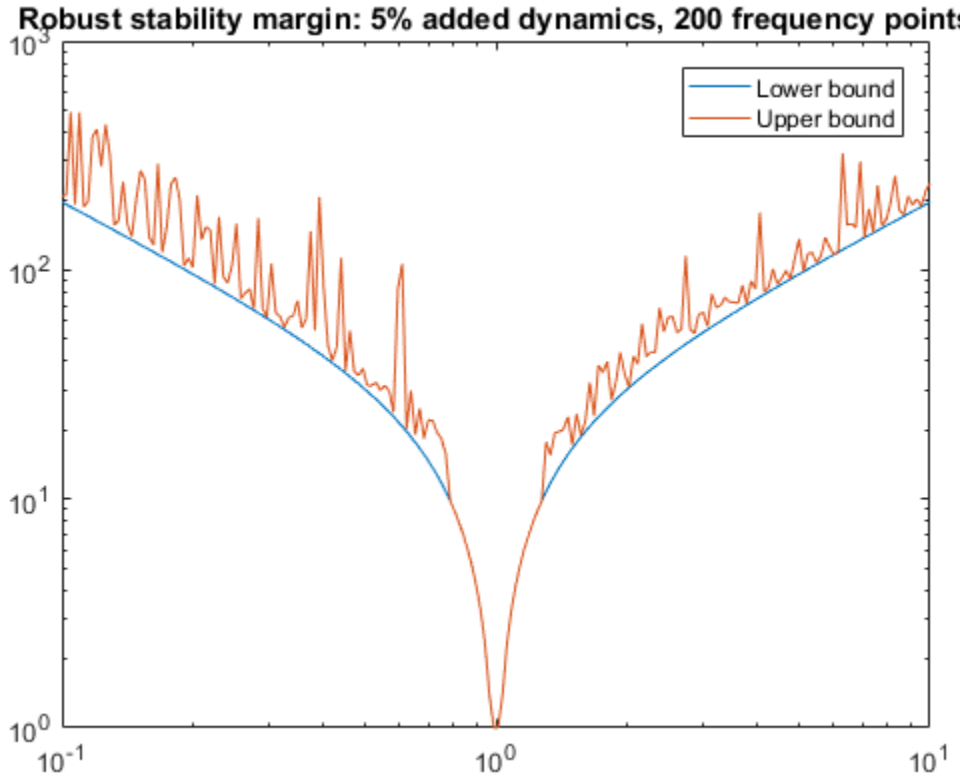
```
OmegaDense = logspace(-1,1,200);
[stabmarg,~,info] = robstab(sysreg,OmegaDense);
stabmarg
```

```
stabmarg =

  struct with fields:

          LowerBound: 1.0026
```

```
        UpperBound: 1.0056
CriticalFrequency: 0.9885
```

Plot the robustness margin as a function of frequency.

```
loglog(OmegaDense,info.Bounds(:,1),OmegaDense,info.Bounds(:,2))
title('Robust stability margin: 5% added dynamics, 200 frequency points')
legend('Lower bound','Upper bound')
```



The computed margin is now close to 1, the true margin for the original problem. In general, the stability margin of the modified problem is less than or equal to that of the original problem. If it is significantly less, then the answer to the question "What is the stability margin?" is very sensitive to the uncertainty model. In this case, we put more

faith in the value that allows for a few percents of unmodeled dynamics. Either way, the stability margin for the modified problem is more trustworthy.

Automated Regularization of Discontinuous Problems

The command `complexify` automates the procedure of replacing a `ureal` with the sum of a `ureal` and `ultidyn`. The analysis above can be repeated using `complexify` obtaining the same results.

```
sysreg = complexify(sys,0.05,'ultidyn');
[stabmarg,~,info] = robstab(sysreg,OmegaDense);
stabmarg
```

```
stabmarg =

  struct with fields:

          LowerBound: 1.0026
          UpperBound: 1.0056
    CriticalFrequency: 0.9885
```

Note that such regularization is only needed when using frequency gridding. Applying `robstab` directly to the original uncertain model `sys` yields the correct margin without frequency gridding or need for regularization.

References

The continuity of the robust stability margin, and the subsequent computational and interpretational difficulties raised by the presence of discontinuities are considered in [1]. The consequences and interpretations of the regularization illustrated in this small example are described in [2]. An extensive analysis of regularization for 2-parameter example is given in [2].

[1] Barmish, B.R., Khargonekar, P.P, Shi, Z.C., and R. Tempo, "Robustness margin need not be a continuous function of the problem data," Systems & Control Letters, Vol. 15, No. 2, 1990, pp. 91-98.

[2] Packard, A., and P. Pandey, "Continuity properties of the real/complex structured singular value," Vol. 38, No. 3, 1993, pp. 415-428.

See Also

`robstab`

Related Examples

- "Robust Stability, Robust Performance and Mu Analysis" on page 2-7

More About

- "Robustness and Worst-Case Analysis" on page 2-2

3

Introduction to Linear Matrix Inequalities

Linear Matrix Inequalities

Linear Matrix Inequalities (LMIs) and LMI techniques have emerged as powerful design tools in areas ranging from control engineering to system identification and structural design. Three factors make LMI techniques appealing:

- A variety of design specifications and constraints can be expressed as LMIs.
- Once formulated in terms of LMIs, a problem can be solved *exactly* by efficient convex optimization algorithms (see "LMI Solvers" on page 4-22).
- While most problems with multiple constraints or objectives lack analytical solutions in terms of matrix equations, they often remain tractable in the LMI framework. This makes LMI-based design a valuable alternative to classical "analytical" methods.

See [9] for a good introduction to LMI concepts. Robust Control Toolbox™ software is designed as an easy and progressive gateway to the new and fast-growing field of LMIs:

- For users who occasionally need to solve LMI problems, the LMI Editor and the tutorial introduction to LMI concepts and LMI solvers provide for quick and easy problem solving.
- For more experienced LMI users, LMI Lab, offers a rich, flexible, and fully programmable environment to develop customized LMI-based tools.

LMI Features

Robust Control Toolbox LMI functionality serves two purposes:

- Provide state-of-the-art tools for the LMI-based analysis and design of robust control systems
- Offer a flexible and user-friendly environment to specify and solve general LMI problems (the LMI Lab)

Examples of LMI-based analysis and design tools include

- Functions to analyze the robust stability and performance of uncertain systems with varying parameters (`popov`, `quadstab`, `quadperf` ...)
- Functions to design robust control with a mix of H_2, H_∞, and pole placement objectives (`h2hinfsyn`)
- Functions for synthesizing robust gain-scheduled H_∞ controllers (`hinfgs`)

For users interested in developing their own applications, the LMI Lab provides a general-purpose and fully programmable environment to specify and solve virtually any LMI problem. Note that the scope of this facility is by no means restricted to control-oriented applications.

Note Robust Control Toolbox software implements state-of-the-art interior-point LMI solvers. While these solvers are significantly faster than classical convex optimization algorithms, you should keep in mind that the complexity of LMI computations can grow quickly with the problem order (number of states). For example, the number of operations required to solve a Riccati equation is $o(n^3)$ where n is the state dimension, while the cost of solving an equivalent "Riccati inequality" LMI is $o(n^6)$.

More About

- "LMIs and LMI Problems" on page 3-4

LMIs and LMI Problems

A linear matrix inequality (LMI) is any constraint of the form

$$A(x) := A_0 + x_1A_1 + ... + x_NA_N < 0$$

where

- $x = (x_1, \ldots, x_N)$ is a vector of unknown scalars (the *decision* or *optimization* variables)
- $A_0, \ldots, A_N$ are given *symmetric* matrices
- < 0 stands for "negative definite," i.e., the largest eigenvalue of $A(x)$ is negative

Note that the constraints $A(x) > 0$ and $A(x) < B(x)$ are special cases of Equation 3-1 since they can be rewritten as $-A(x) < 0$ and $A(x) - B(x) < 0$, respectively.

The LMI of Equation 3-1 is a convex constraint on x since $A(y) < 0$ and $A(z) < 0$ imply that $A\left(\frac{y+z}{2}\right) < 0$. As a result,

- Its solution set, called the *feasible set*, is a convex subset of R^N
- Finding a solution x to Equation 3-1, if any, is a convex optimization problem.

Convexity has an important consequence: even though Equation 3-1 has no analytical solution in general, it can be solved numerically with guarantees of finding a solution when one exists. Note that a system of LMI constraints can be regarded as a single LMI since

$$\begin{cases} A_1(x) < 0 \\ \quad \vdots \\ A_K(x) < 0 \end{cases}$$

is equivalent to

$$A(x) := \mathrm{diag}\left(\mathrm{A}_1(x), \ldots, \mathrm{A}_\mathrm{K}(x)\right) < 0$$

where diag $(A_1(x), \ldots, A_\mathrm{K}(x))$ denotes the block-diagonal matrix with

$A_1(x)$, . . . , $A_K(x)$ on its diagonal. Hence multiple LMI constraints can be imposed on the vector of decision variables x without destroying convexity.

In most control applications, LMIs do not naturally arise in the canonical form of Equation 3-1 , but rather in the form

$$L(X_1, \ldots, X_n) < R(X_1, \ldots, X_n)$$

where $L(.)$ and $R(.)$ are affine functions of some structured *matrix* variables X_1, . . . , X_n. *A* simple example is the Lyapunov inequality

$$A^T X + XA < 0$$

where the unknown X is a symmetric matrix. Defining x_1, . . . , x_N as the independent scalar entries of X, this LMI could be rewritten in the form of Equation 3-1. Yet it is more convenient and efficient to describe it in its natural form Equation 3-2, which is the approach taken in the LMI Lab.

Related Examples

- "LMI Applications" on page 3-6

LMI Applications

Finding a solution x to the LMI system

$A(x) < 0$

is called the *feasibility problem*. Minimizing a convex objective under LMI constraints is also a convex problem. In particular, the *linear objective minimization problem*:

Minimize $c^T x$ subject to

$A(x) < 0$

plays an important role in LMI-based design. Finally, the *generalized eigenvalue minimization problem*

Minimize λ subject to

$$A(x) < \lambda B(x)$$
$$B(x) > 0$$
$$C(x) > 0$$

is quasi-convex and can be solved by similar techniques. It owes its name to the fact that is related to the largest generalized eigenvalue of the pencil $(A(x),B(x))$.

Many control problems and design specifications have LMI formulations [9]. This is especially true for Lyapunov-based analysis and design, but also for optimal LQG control, H^∞ control, covariance control, etc. Further applications of LMIs arise in estimation, identification, optimal design, structural design [6], [7], matrix scaling problems, and so on. The main strength of LMI formulations is the ability to combine various design constraints or objectives in a numerically tractable manner.

A nonexhaustive list of problems addressed by LMI techniques includes the following:

- Robust stability of systems with LTI uncertainty (μ-analysis) ([24], [21], [27])
- Robust stability in the face of sector-bounded nonlinearities (Popov criterion) ([22], [28], [13], [16])
- Quadratic stability of differential inclusions ([15], [8])
- Lyapunov stability of parameter-dependent systems ([12])

- Input/state/output properties of LTI systems (invariant ellipsoids, decay rate, etc.) ([9])
- Multi-model/multi-objective state feedback design ([4], [17], [3], [9], [10])
- Robust pole placement
- Optimal LQG control ([9])
- Robust H^∞ control ([11], [14])
- Multi-objective H^∞ synthesis ([18], [23], [10], [18])
- Design of robust gain-scheduled controllers ([5], [2])
- Control of stochastic systems ([9])
- Weighted interpolation problems ([9])

To hint at the principles underlying LMI design, let's review the LMI formulations of a few typical design objectives.

Stability

The stability of the dynamic system

$$\dot{x} = Ax$$

is equivalent to the feasibility of the following problem:

Find $P = P^T$ such that $A^T P + P A < 0$, $P > I$.

This can be generalized to linear differential inclusions (LDI)

$$\dot{x} = A(t)x$$

where $A(t)$ varies in the convex envelope of a set of LTI models:

$$A(t) \in \mathrm{Co}\{A_1, \ldots, A_n\} = \left\{ \sum_{i=1}^{n} a_i A_i : a_i \geq 0, \sum_{i=1}^{N} a_i = 1 \right\}.$$

A sufficient condition for the asymptotic stability of this LDI is the feasibility of

Find $P = P^T$ such that $A_i^T P + P A_i < 0, \quad P > I$.

RMS Gain

The random-mean-squares (RMS) gain of a stable LTI system

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

is the largest input/output gain over all bounded inputs $u(t)$. This gain is the global minimum of the following linear objective minimization problem [1], [25], [26].

Minimize γ over $X = X^T$ and γ such that

$$\begin{pmatrix} A^T X + XA & XB & C^T \\ B^T X & -\gamma I & D^T \\ C & D & -\gamma I \end{pmatrix} < 0$$

and

$$X > 0.$$

LQG Performance

For a stable LTI system

$$G \begin{cases} \dot{x} = Ax + Bw \\ y = Cx \end{cases}$$

where w is a white noise disturbance with unit covariance, the LQG or H_2 performance $\|G\|_2$ is defined by

$$\begin{aligned} \|G\|_2^2 &:= \lim_{T \to \infty} E\left\{ \frac{1}{T} \int_0^T y^T(t) y(t) dt \right\} \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} G^H(j\omega) G(j\omega) d\omega. \end{aligned}$$

It can be shown that

$$\|G\|_2^2 = \inf\left\{\mathrm{Trace}\left(\mathrm{CPC}^T\right) : AP + PA^T + BB^T < 0\right\}.$$

Hence $\|G\|_2^2$ is the global minimum of the LMI problem. Minimize Trace (Q) over the symmetric matrices P,Q such that

$$AP + PA^T + BB^T < 0$$

and

$$\begin{pmatrix} Q & CP \\ PC^T & P \end{pmatrix} > 0.$$

Again this is a linear objective minimization problem since the objective Trace (Q) is linear in the decision variables (free entries of P,Q).

More About

- "Tools for Specifying and Solving LMIs" on page 4-2

Further Mathematical Background

Efficient interior-point algorithms are now available to solve the three generic LMI problems Equation 3-2–Equation 3-4 defined in "LMI Applications" on page 3-6. These algorithms have a polynomial-time complexity. That is, the number $N(\varepsilon)$ of flops needed to compute an ε-accurate solution is bounded by

$$M N^3 \log(V/\varepsilon)$$

where M is the total row size of the LMI system, N is the total number of scalar decision variables, and V is a data-dependent scaling factor. Robust Control Toolbox software implements the Projective Algorithm of Nesterov and Nemirovski [20], [19]. In addition to its polynomial-time complexity, this algorithm does not require an initial feasible point for the linear objective minimization problem Equation 3-3or the generalized eigenvalue minimization problem Equation 3-4.

Some LMI problems are formulated in terms of inequalities rather than strict inequalities. For instance, a variant of Equation 3-3 is

Minimize $c^T x$ subject to $A(x) < 0$.

While this distinction is immaterial in general, it matters when $A(x)$ can be made negative semi-definite but not negative definite. A simple example is:

Minimize $c^T x$ subject to

$$\begin{pmatrix} x & x \\ x & x \end{pmatrix} \geq 0.$$

Such problems cannot be handled directly by interior-point methods which require strict feasibility of the LMI constraints. A well-posed reformulation of Equation 3-5 would be

Minimize $c^T x$ subject to $x \geq 0$.

Keeping this subtlety in mind, we always use strict inequalities in this manual.

Related Examples

- "LMI Applications" on page 3-6

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LMI Lab

Tools for Specifying and Solving LMIs

The LMI Lab is a high-performance package for solving general LMI problems. It blends simple tools for the specification and manipulation of LMIs with powerful LMI solvers for three generic LMI problems. Thanks to a structure-oriented representation of LMIs, the various LMI constraints can be described in their natural block-matrix form. Similarly, the optimization variables are specified directly as *matrix variables* with some given structure. Once an LMI problem is specified, it can be solved numerically by calling the appropriate LMI solver. The three solvers `feasp`, `mincx`, and `gevp` constitute the computational engine of the LMI portion of Robust Control Toolbox software. Their high performance is achieved through C-MEX implementation and by taking advantage of the particular structure of each LMI.

The LMI Lab offers tools to

- Specify LMI systems either symbolically with the LMI Editor or incrementally with the `lmivar` and `lmiterm` commands
- Retrieve information about existing systems of LMIs
- Modify existing systems of LMIs
- Solve the three generic LMI problems (feasibility problem, linear objective minimization, and generalized eigenvalue minimization)
- Validate results

This chapter gives a tutorial introduction to the LMI Lab as well as more advanced tips for making the most out of its potential.

Some Terminology

Any linear matrix inequality can be expressed in the canonical form
$L(x) = L_0 + x_1L_1 + \ldots + x_NL_N < 0$

where

- $L_0, L_1, \ldots, L_N$ are given symmetric matrices
- $x = (x_1, \ldots, x_N)^T \in R^N$ is the vector of scalar variables to be determined. We refer to $x_1, \ldots, x_N$ as the *decision variables*. The names “design variables” and “optimization variables” are also found in the literature.

Even though this canonical expression is generic, LMIs rarely arise in this form in control applications. Consider for instance the Lyapunov inequality

$$A^T X + XA < 0$$

where

$$A = \begin{pmatrix} -1 & 2 \\ 0 & -2 \end{pmatrix}$$

and the variable

$$X = \begin{pmatrix} x_1 & x_2 \\ x_2 & x_3 \end{pmatrix}$$

is a symmetric matrix. Here the decision variables are the free entries x_1, x_2, x_3 of X and the canonical form of this LMI reads

$$x_1 \begin{pmatrix} -2 & 2 \\ 2 & 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 & -3 \\ -3 & 4 \end{pmatrix} + x_3 \begin{pmatrix} 0 & 0 \\ 0 & -4 \end{pmatrix} < 0.$$

Clearly this expression is less intuitive and transparent than Equation 4-1. Moreover, the number of matrices involved in Equation 4-2 grows roughly as $n^2/2$ if n is the size of the A matrix. Hence, the canonical form is very inefficient from a storage viewpoint since it requires storing $o(n^2/2)$ matrices of size n when the single n-by-n matrix A would be sufficient. Finally, working with the canonical form is also detrimental to the efficiency of the LMI solvers. For these various reasons, the LMI Lab uses a *structured representation* of LMIs. For instance, the expression $A^TX + XA$ in the Lyapunov inequality Equation 4-1 is explicitly described as a function of the matrix variable X, and only the A matrix is stored.

In general, LMIs assume a block matrix form where each block is an affine combination of the matrix variables. As a fairly typical illustration, consider the following LMI drawn from H^∞ theory

$$N^T \begin{pmatrix} A^T X + XA & XC^T & B \\ CX & -\gamma I & D \\ B^T & D^T & -\gamma I \end{pmatrix} N < 0$$

where A, B, C, D, and N are given matrices and the problem variables are $X = X^T \in R^{n\times n}$ and $\gamma \in R$. We use the following terminology to describe such LMIs:

- N is called the *outer factor*, and the block matrix

$$L(X,\gamma) = \begin{pmatrix} A^T X + XA & XC^T & B \\ CX & -\gamma I & D \\ B^T & D^T & -\gamma I \end{pmatrix}$$

  is called the *inner factor*. The outer factor *needs not be square* and is *often absent*.
- X and γ are the *matrix variables* of the problem. Note that scalars are considered as 1-by-1 matrices.
- The inner factor $L(X, \gamma)$ is a symmetric *block matrix*, its block structure being characterized by the sizes of its diagonal blocks. By symmetry, $L(X, \gamma)$ is entirely specified by the blocks on or above the diagonal.
- Each block of $L(X, \gamma)$ is an affine expression in the matrix variables X and γ. This expression can be broken down into a sum of elementary *terms*. For instance, the block (1,1) contains two elementary terms: $A^T X$ and XA.
- Terms are either *constant* or *variable*. Constant terms are fixed matrices like B and D above. Variable terms involve one of the matrix variables, like XA, XC^T, and $-\gamma I$ above.

The LMI (Equation 4-3) is specified by the list of terms in each block, as is any LMI regardless of its complexity.

As for the matrix variables X and γ, they are characterized by their dimensions and structure. Common structures include rectangular unstructured, symmetric, skew-symmetric, and scalar. More sophisticated structures are sometimes encountered in control problems. For instance, the matrix variable X could be constrained to the block-diagonal structure:

$$X = \left(\begin{array}{c|cc} x_1 & 0 & 0 \\ \hline 0 & x_2 & x_3 \\ 0 & x_3 & x_4 \end{array}\right).$$

Another possibility is the symmetric Toeplitz structure:

$$X = \begin{pmatrix} x_1 & x_2 & x_3 \\ x_2 & x_1 & x_2 \\ x_3 & x_2 & x_1 \end{pmatrix}.$$

Summing up, structured LMI problems are specified by declaring the matrix variables and describing the term content of each LMI. This term-oriented description is systematic and accurately reflects the specific structure of the LMI constraints. There is no built-in limitation on the number of LMIs that you can specify or on the number of blocks and terms in any given LMI. LMI systems of arbitrary complexity can therefore, be defined in the LMI Lab.

Overview of the LMI Lab

The LMI Lab offers tools to specify, manipulate, and numerically solve LMIs. Its main purpose is to

- Allow for straightforward description of LMIs in their natural block-matrix form
- Provide easy access to the LMI solvers (optimization codes)
- Facilitate result validation and problem modification

The structure-oriented description of a given LMI system is stored as a single vector called the *internal representation* and generically denoted by `LMISYS` in the sequel. This vector encodes the structure and dimensions of the LMIs and matrix variables, a description of all LMI terms, and the related numerical data. It must be stressed that you need not attempt to read or understand the content of `LMISYS` since all manipulations involving this internal representation can be performed in a transparent manner with LMI-Lab tools.

The LMI Lab supports the following functionalities:

Specification of a System of LMIs

LMI systems can be either specified as symbolic matrix expressions with the interactive graphical user interface `lmiedit`, or assembled incrementally with the two commands `lmivar` and `lmiterm`. The first option is more intuitive and transparent while the second option is more powerful and flexible.

Information Retrieval

The interactive function `lmiinfo` answers qualitative queries about LMI systems created with `lmiedit` or `lmivar` and `lmiterm`. You can also use `lmiedit` to visualize the LMI system produced by a particular sequence of `lmivar`/`lmiterm` commands.

Solvers for LMI Optimization Problems

General-purpose LMI solvers are provided for the three generic LMI problems defined in "LMI Applications" on page 3-6. These solvers can handle very general LMI systems and matrix variable structures. They return a feasible or optimal vector of decision variables x^*. The corresponding values $X_1^*,\ldots,X_K^*$ of the matrix variables are given by the function `dec2mat`.

Result Validation

The solution x^* produced by the LMI solvers is easily validated with the functions `evallmi` and `showlmi`. This allows a fast check and/or analysis of the results. With `evallmi`, all variable terms in the LMI system are evaluated for the value x^* of the decision variables. The left and right sides of each LMI then become constant matrices that can be displayed with `showlmi`.

Modification of a System of LMIs

An existing system of LMIs can be modified in two ways:

- An LMI can be removed from the system with `dellmi`.
- A matrix variable X can be deleted using `delmvar`. It can also be instantiated, that is, set to some given matrix value. This operation is performed by `setmvar` and allows, for example, to fix some variables and solve the LMI problem with respect to the remaining ones.

Related Examples

- "Specifying a System of LMIs" on page 4-7

Specifying a System of LMIs

The LMI Lab can handle any system of LMIs of the form

$$N^T L(X_1, \ldots, X_K)\, N < MT\, R(X_1, \ldots, X_K)\, M$$

where

- $X_1, \ldots, X_K$ are matrix variables with some prescribed structure
- The left and right outer factors N and M are given matrices with *identical* dimensions
- The left and right inner factors $L(.)$ and $R(.)$ are symmetric block matrices with identical block structures, each block being an affine combination of $X_1, \ldots, X_K$ and their transposes.

Note Throughout this chapter, "left side" refers to what is on the "smaller" side of the inequality, and "right side" to what is on the "larger" side. Accordingly, X is called the right-hand side and 0 the left side of the LMI

$$0 < X$$

even when this LMI is written as $X > 0$.

The specification of an LMI system involves two steps:

1. Declare the dimensions and structure of each matrix variable $X_1, \ldots, X_K$.
2. Describe the term content of each LMI.

This process creates the so-called *internal representation* of the LMI system. This computer description of the problem is used by the LMI solvers and in all subsequent manipulations of the LMI system. It is stored as a single vector called `LMISYS`.

There are two ways of generating the internal description of a given LMI system: (1) by a sequence of `lmivar`/`lmiterm` commands that build it incrementally, or (2) via the LMI Editor `lmiedit` where LMIs can be specified directly as symbolic matrix expressions. Though somewhat less flexible and powerful than the command-based description, the LMI Editor is more straightforward to use, hence particularly well-suited for beginners. Thanks to its coding and decoding capabilities, it also constitutes a good tutorial introduction to `lmivar` and `lmiterm`. Accordingly, beginners may elect to skip the subsections on `lmivar` and `lmiterm` and to concentrate on the GUI-based specification of LMIs with `lmiedit`.

Related Examples

- “Specify LMI System at the Command Line” on page 4-9
- “Specify LMIs with the LMI Editor GUI” on page 4-16

Specify LMI System at the Command Line

The following tutorial example is used to illustrate the specification of LMI systems with the LMI Lab tools.

Specifying LMI System

Consider a stable transfer function

$$G(s) = C(sI - A)^{-1} B$$

with four inputs, four outputs, and six states, and consider the set of input/output scaling matrices D with block-diagonal structure

$$D = \begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_1 & 0 & 0 \\ 0 & 0 & d_2 & d_3 \\ 0 & 0 & d_4 & d_5 \end{pmatrix}.$$

The following problem arises in the robust stability analysis of systems with time-varying uncertainty [4]:

Find, if any, a scaling D of structure (Equation 4-5) such that the largest gain across frequency of $D\,G(s)\,D^{-1}$ is less than one.

This problem has a simple LMI formulation: there exists an adequate scaling D if the following feasibility problem has solutions:

Find two symmetric matrices $X \in R^{6\times 6}$ and $S = D^T D \in R^{4\times 4}$ such that

$$\begin{pmatrix} A^T X + XA + C^T SC & XB \\ B^T X & -S \end{pmatrix} < 0$$

$$X > 0$$

$$S > 1.$$

The LMI system (Equation 4-6, Equation 4-7, and Equation 4-8) can be described with the LMI Editor as outlined below. Alternatively, its internal description can be generated with `lmivar` and `lmiterm` commands as follows:

```
setlmis([])
X=lmivar(1,[6 1])
S=lmivar(1,[2 0;2 1])

% 1st LMI
lmiterm([1 1 1 X],1,A,'s')
lmiterm([1 1 1 S],C',C)
lmiterm([1 1 2 X],1,B)
lmiterm([1 2 2 S],-1,1)

% 2nd LMI
lmiterm([-2 1 1 X],1,1)

% 3rd LMI
lmiterm([-3 1 1 S],1,1)
lmiterm([3 1 1 0],1)

LMISYS = getlmis
```

Here the `lmivar` commands define the two matrix variables X and S while the `lmiterm` commands describe the various terms in each LMI. Upon completion, `getlmis` returns the internal representation `LMISYS` of this LMI system. The following subsections give more details on the syntax and usage of these various commands:

- "Initializing the LMI System" on page 4-10
- "Specifying the LMI Variables" on page 4-11
- "Specifying Individual LMIs" on page 4-13

More information on how the internal representation is updated by `lmivar`/`lmiterm` can also be found in "How lmivar and lmiterm Manage LMI Representation" on page 4-20.

Initializing the LMI System

The description of an LMI system should begin with `setlmis` and end with `getlmis`. The function `setlmis` initializes the LMI system description. When specifying a new system, type

```
setlmis([])
```

To add on to an existing LMI system with internal representation `LMISO`, type

```
setlmis(LMISO)
```

Specifying the LMI Variables

The matrix variables are declared one at a time with `lmivar` and are characterized by their structure. To facilitate the specification of this structure, the LMI Lab offers two predefined structure types along with the means to describe more general structures:

Type 1	Symmetric block diagonal structure. This corresponds to matrix variables of the form $$X = \begin{pmatrix} D_1 & 0 & \dots & 0 \\ 0 & D_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & D_r \end{pmatrix}$$ where each diagonal block D_j is square and is either zero, a *full symmetric* matrix, or a *scalar* matrix $D_j = d \times I, \ d \in R$ This type encompasses ordinary symmetric matrices (single block) and scalar variables (one block of size one).
Type 2	**Rectangular** structure. This corresponds to arbitrary rectangular matrices without any particular structure.
Type 3	**General** structures. This third type is used to describe more sophisticated structures and/or correlations between the matrix variables. The principle is as follows: each entry of X is specified independently as either 0, xn, or $-x$n where xn denotes the n-th decision variable in the problem. For details on how to use Type 3, see "Structured Matrix Variables" on page 4-33 as well as the `lmivar` entry in the reference pages.

In "Specifying LMI System" on page 4-9, the matrix variables X and S are of Type 1. Indeed, both are symmetric and S inherits the block-diagonal structure from Equation 4-5 of D. Specifically, S is of the form

$$S = \begin{pmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_1 & 0 & 0 \\ 0 & 0 & s_2 & s_3 \\ 0 & 0 & s_3 & s_4 \end{pmatrix}.$$

After initializing the description with the command `setlmis([])`, these two matrix variables are declared by

```
lmivar(1,[6 1]) % X
lmivar(1,[2 0;2 1]) % S
```

In both commands, the first input specifies the structure type and the second input contains additional information about the structure of the variable:

- For a matrix variable X of Type 1, this second input is a matrix with two columns and as many rows as diagonal blocks in X. The first column lists the sizes of the diagonal blocks and the second column specifies their nature with the following convention:

  1: full symmetric block

  0: scalar block

  –1: zero block

  In the second command, for instance, `[2 0;2 1]` means that S has two diagonal blocks, the first one being a 2-by-2 scalar block and the second one a 2-by-2 full block.
- For matrix variables of Type 2, the second input of `lmivar` is a two-entry vector listing the row and column dimensions of the variable. For instance, a 3-by-5 rectangular matrix variable would be defined by

  ```
  lmivar(2,[3 5])
  ```

For convenience, `lmivar` also returns a "tag" that identifies the matrix variable for subsequent reference. For instance, X and S in "Specifying LMI System" on page 4-9 could be defined by

```
X = lmivar(1,[6 1])
S = lmivar(1,[2 0;2 1])
```

The identifiers X and S are integers corresponding to the ranking of X and S in the list of matrix variables (in the order of declaration). Here their values would be `X=1` and `S=2`.

Note that these identifiers still point to X and S after deletion or instantiation of some of the matrix variables. Finally, `lmivar` can also return the total number of decision variables allocated so far as well as the entry-wise dependence of the matrix variable on these decision variables (see the `lmivar` entry in the reference pages for more details).

Specifying Individual LMIs

After declaring the matrix variables with `lmivar`, we are left with specifying the term content of each LMI. Recall that LMI terms fall into three categories:

- The *constant terms*, i.e., fixed matrices like I in the left side of the LMI $S > I$
- The *variable terms*, i.e., terms involving a matrix variable. For instance, A^TX and C^TSC in Equation 4-6. Variable terms are of the form PXQ where X is a variable and P, Q are given matrices called the left and right *coefficients*, respectively.
- The *outer factors*

The following rule should be kept in mind when describing the term content of an LMI:

Note: Specify only the terms in the blocks on or above the diagonal. The inner factors being symmetric, this is sufficient to specify the entire LMI. *Specifying all blocks results in the duplication of off-diagonal terms, hence in the creation of a different LMI.* Alternatively, you can describe the blocks on or below the diagonal.

LMI terms are specified one at a time with `lmiterm`. For instance, the LMI

$$\begin{pmatrix} A^TX + XA + C^TSC & XB \\ B^TX & -S \end{pmatrix} < 0$$

is described by

```
lmiterm([1 1 1 1],1,A,'s')
lmiterm([1 1 1 2],C',C)
lmiterm([1 1 2 1],1,B)
lmiterm([1 2 2 2],-1,1)
```

These commands successively declare the terms $A^TX + XA$, C^TSC, XB, and $-S$. In each command, the first argument is a four-entry vector listing the term characteristics as follows:

- The first entry indicates to which LMI the term belongs. The value `m` means "left side of the m-th LMI," and `-m` means "right side of the m-th LMI."
- The second and third entries identify the block to which the term belongs. For instance, the vector `[1 1 2 1]` indicates that the term is attached to the (1, 2) block.
- The last entry indicates which matrix variable is involved in the term. This entry is `0` for constant terms, `k` for terms involving the k-th matrix variable X_k, and `-k` for terms involving X_k^T (here X and S are first and second variables in the order of declaration).

Finally, the second and third arguments of `lmiterm` contain the numerical data (values of the constant term, outer factor, or matrix coefficients P and Q for variable terms PXQ or PX^TQ). These arguments must refer to existing MATLAB variables and be *real-valued*. See "Complex-Valued LMIs" on page 4-35 for the specification of LMIs with complex-valued coefficients.

Some shorthand is provided to simplify term specification. First, blocks are zero by default. Second, in *diagonal blocks* the extra argument `'s'` allows you to specify the conjugated expression $AXB + B^TX^TA^T$ with a *single* `lmiterm` command. For instance, the first command specifies $A^TX + XA$ as the "symmetrization" of XA. Finally, scalar values are allowed as shorthand for s*calar matrices*, i.e., matrices of the form αI with α scalar. Thus, a constant term of the form αI can be specified as the "scalar" α. This also applies to the coefficients P and Q of variable terms. The dimensions of scalar matrices are inferred from the context and set to 1 by default. For instance, the third LMI $S > I$ in "Example: Specifying Matrix Variable Structures" on page 4-33 is described by

```
lmiterm([-3 1 1 2],1,1)      % 1*S*1 = S
lmiterm([3 1 1 0],1)         % 1*I = I
```

Recall that by convention `S` is considered as the right side of the inequality, which justifies the –3 in the first command.

Finally, to improve readability it is often convenient to attach an identifier (tag) to each LMI and matrix variable. The variable identifiers are returned by `lmivar` and the LMI identifiers are set by the function `newlmi`. These identifiers can be used in `lmiterm` commands to refer to a given LMI or matrix variable. For the LMI system of "Specifying LMI System" on page 4-9, this would look like:

```
setlmis([])
X = lmivar(1,[6 1])
S = lmivar(1,[2 0;2 1])
```

```
BRL = newlmi
lmiterm([BRL 1 1 X],1,A,'s')
lmiterm([BRL 1 1 S],C',C)
lmiterm([BRL 1 2 X],1,B)
lmiterm([BRL 2 2 S],-1,1)

Xpos = newlmi
lmiterm([-Xpos 1 1 X],1,1)

Slmi = newlmi
lmiterm([-Slmi 1 1 S],1,1)
lmiterm([Slmi 1 1 0],1)
```

When the LMI system is completely specified, type

```
LMISYS = getlmis
```

This returns the internal representation `LMISYS` of this LMI system. This MATLAB description of the problem can be forwarded to other LMI-Lab functions for subsequent processing. The command `getlmis` must be used *only once* and after declaring *all* matrix variables and LMI terms.

Here the identifiers `X` and `S` point to the variables X and S while the tags `BRL`, `Xpos`, and `Slmi` point to the first, second, and third LMI, respectively. Note that `-Xpos` refers to the right-hand side of the second LMI. Similarly, `-X` would indicate transposition of the variable X.

Related Examples

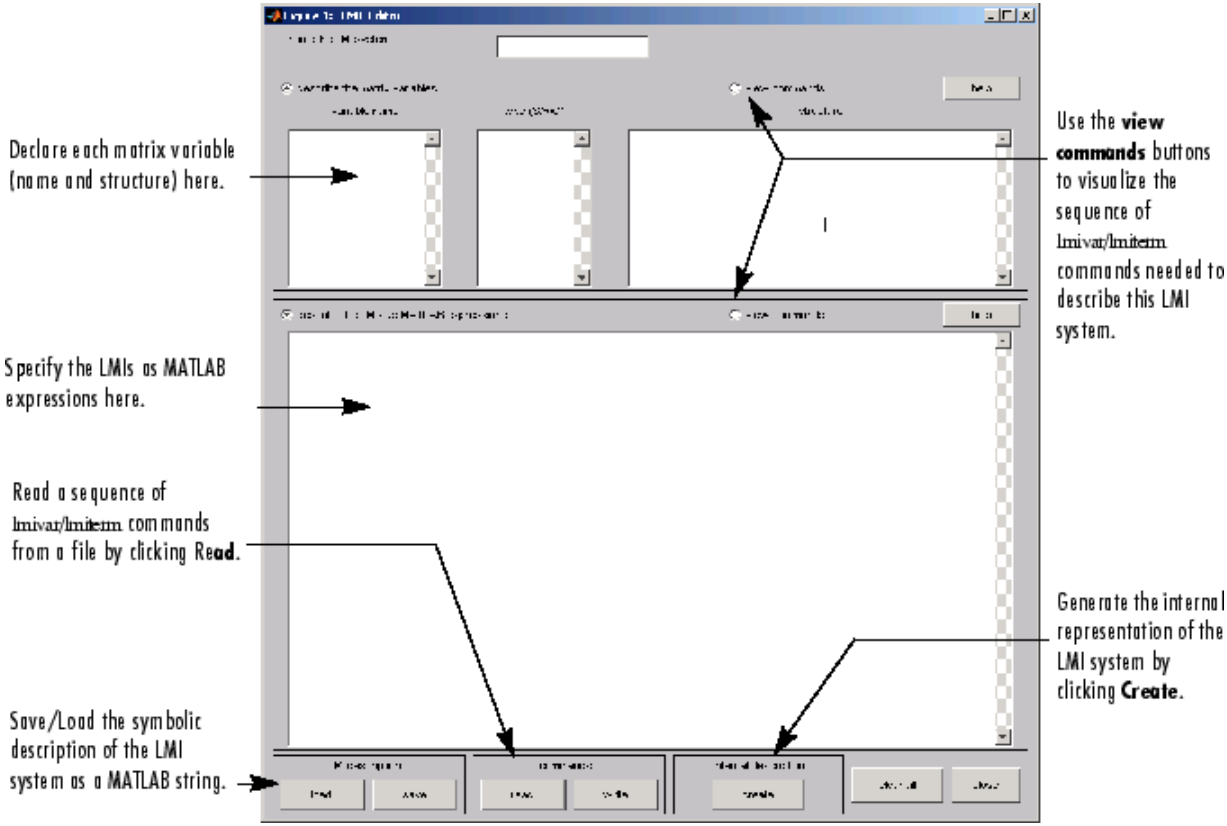
- “Specify LMIs with the LMI Editor GUI” on page 4-16

Specify LMIs with the LMI Editor GUI

The LMI Editor `lmiedit` is a graphical user interface (GUI) to specify LMI systems in a straightforward symbolic manner. Typing

```
lmiedit
```

calls up a window with several editable text areas and various buttons.



In more detail, to specify your LMI system,

1. Declare each matrix variable (name and structure) in the upper half of the worksheet. The structure is characterized by its type (`S` for symmetric block diagonal, `R` for unstructured, and `G` for other structures) and by an additional " structure" matrix. This matrix contains specific information about the structure and

corresponds to the second argument of `lmivar` (see "Specifying the LMI Variables" on page 4-11 for details).

Please use *one line per matrix variable* in the text editing areas.

2 Specify the LMIs as MATLAB expressions in the lower half of the worksheet. For instance, the LMI

$$\begin{pmatrix} A^T X + XA & XB \\ B^T X & -I \end{pmatrix} < 0$$

is entered by typing

```
[a'*x+x*a x*b; b'*x -1] < 0
```

if `x` is the name given to the matrix variable X in the upper half of the worksheet. The left- and right-hand sides of the LMIs should be *valid* MATLAB expressions.

Once the LMI system is fully specified, the following tasks can be performed by clicking the corresponding button:

- Visualize the sequence of `lmivar`/`lmiterm` commands needed to describe this LMI system (`view commands` button). Conversely, the LMI system defined by a particular sequence of `lmivar`/`lmiterm` commands can be displayed as a MATLAB expression by clicking on the **describe...** buttons.

  Beginners can use this facility as a tutorial introduction to the `lmivar` and `lmiterm` commands.

- Save the symbolic description of the LMI system (`save` button). This description can be reloaded later on by clicking the **load** button.

- Read a sequence of `lmivar`/`lmiterm` commands from a file (**read** button). You can then click on **describe the matrix variables** or **describe the LMIs** to visualize the symbolic expression of the LMI system specified by these commands. The file should describe a single LMI system but may otherwise contain any sequence of MATLAB commands.

  This feature is useful for code validation and debugging.

  Write in a file the sequence of `lmivar`/`lmiterm` commands needed to describe a particular LMI system (**write** button).

This is helpful to develop code and prototype MATLAB functions based on the LMI Lab.

- Generate the internal representation of the LMI system by clicking **create**. The result is written in a MATLAB variable named after the LMI system (if the "name of the LMI system" is set to `mylmi`, the internal representation is written in the MATLAB variable `mylmi`). Note that all LMI-related data should be defined in the MATLAB workspace at this stage.

  The internal representation can be passed directly to the LMI solvers or any other LMI Lab function.

Keyboard Shortcuts

As with `lmiterm`, you can use various shortcuts when entering LMI expressions at the keyboard. For instance, zero blocks can be entered simply as `0` and need not be dimensioned. Similarly, the identity matrix can be entered as `1` without dimensioning. Finally, *upper diagonal* LMI blocks need not be fully specified. Rather, you can just type (`*`) in place of each such block.

Limitations

Though fairly general, `lmiedit` is not as flexible as `lmiterm` and the following limitations should be kept in mind:

- Parentheses cannot be used around matrix variables. For instance, the expression

  ```
  (a*x+b)'*c + c'*(a*x+b)
  ```

  is invalid when *x* is a variable name. By contrast,

  ```
  (a+b)'*x + x'*(a+b)
  ```

  is perfectly valid.
- Loops and `if` statements are ignored.
- When turning `lmiterm` commands into a symbolic description of the LMI system, an error is issued if the first argument of `lmiterm` cannot be evaluated. Use the LMI and variable identifiers supplied by `newlmi` and `lmivar` to avoid such difficulties.

Related Examples

- "Specify LMI System at the Command Line" on page 4-9

How lmivar and lmiterm Manage LMI Representation

Users familiar with MATLAB may wonder how `lmivar` and `lmiterm` physically update the internal representation `LMISYS` since `LMISYS` is not an argument to these functions. In fact, all updating is performed through global variables for maximum speed. These global variables are initialized by `setlmis`, cleared by `getlmis`, and are not visible in the workspace. Even though this artifact is transparent from the user's viewpoint, be sure to:

- Invoke `getlmis` only once and after completely specifying the LMI system.
- Refrain from using the command `clear global` before the LMI system description is ended with `getlmis`.

More About

- “Querying the LMI System Description” on page 4-21

Querying the LMI System Description

Recall that the full description of an LMI system is stored as a single vector called the internal representation. The user should not attempt to read or retrieve information directly from this vector. Robust Control Toolbox software provides three functions called `lmiinfo`, `lminbr`, and `matnbr` to extract and display all relevant information in a user-readable format.

lmiinfo

`lminbr` is an interactive facility to retrieve qualitative information about LMI systems. This includes the number of LMIs, the number of matrix variables and their structure, the term content of each LMI block, etc. To invoke `lmiinfo`, enter

```
lmiinfo(LMISYS)
```

where `LMISYS` is the internal representation of the LMI system produced by `getlmis`.

lminbr and matnbr

These two functions return the number of LMIs and the number of matrix variables in the system. To get the number of matrix variables, for instance, enter

```
matnbr(LMISYS)
```

More About

- “LMI Solvers” on page 4-22

LMI Solvers

LMI solvers are provided for the following three generic optimization problems (here x denotes the vector of decision variables, i.e., of the free entries of the matrix variables $X_1, \ldots, X_K$):

- Feasibility problem

  Find $x \in R^N$ (or equivalently matrices $X_1, \ldots, X_K$ with prescribed structure) that satisfies the LMI system

  $A(x) < B(x)$

  The corresponding solver is called `feasp`.

- Minimization of a linear objective under LMI constraints

  Minimize $c^T x$ over $x \in R^N$ subject to $A(x) < B(x)$

  The corresponding solver is called `mincx`.

- Generalized eigenvalue minimization problem

  Minimize λ over $x \in R^N$ subject to

  $$C(x) < D(x)$$

  $$0 < B(x)$$

  $$A(x) < \lambda B(x).$$

  The corresponding solver is called `gevp`.

Note that $A(x) < B(x)$ above is a shorthand notation for general structured LMI systems with decision variables $x = (x_1, \ldots, x_N)$.

The three LMI solvers `feasp`, `mincx`, and `gevp` take as input the internal representation `LMISYS` of an LMI system and return a feasible or optimizing value x^* of the decision variables. The corresponding values of the matrix variables $X_1, \ldots, X_K$ are derived from x^* with the function `dec2mat`. These solvers are C-MEX implementations of the polynomial-time Projective Algorithm Projective Algorithm of Nesterov and Nemirovski [3], [2].

For generalized eigenvalue minimization problems, it is necessary to distinguish between the standard LMI constraints $C(x) < D(x)$ and the linear-fractional LMIs

$A(x) < \lambda B(x)$

attached to the minimization of the generalized eigenvalue λ. When using `gevp`, you should follow these three rules to ensure proper specification of the problem:

- Specify the LMIs involving λ as $A(x) < B(x)$ (*without* the λ)
- Specify them *last* in the LMI system. `gevp` systematically assumes that the last L LMIs are linear-fractional if L is the number of LMIs involving λ
- Add the constraint $0 < B(x)$ or any other constraint that enforces it. This positivity constraint is required for well-posedness of the problem and is not automatically added by `gevp`.

An initial guess `xinit` for x can be supplied to `mincx` or `gevp`. Use `mat2dec` to derive `xinit` from given values of the matrix variables $X_1, \ldots, X_K$.

The example "Minimize Linear Objectives under LMI Constraints" on page 4-24 illustrates the use of the `mincx` solver.

Related Examples

- "Minimize Linear Objectives under LMI Constraints" on page 4-24

Minimize Linear Objectives under LMI Constraints

Consider the optimization problem:

Minimize Trace(X) subject to

$$A^TX + XA + XBB^TX + Q < 0$$

with data

$$A = \begin{pmatrix} -1 & -2 & 1 \\ 3 & 2 & 1 \\ 1 & -2 & -1 \end{pmatrix}; \quad B = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}; \quad Q = \begin{pmatrix} 1 & -1 & 0 \\ -1 & -3 & -12 \\ 0 & -12 & -36 \end{pmatrix}.$$

It can be shown that the minimizer X^* is simply the stabilizing solution of the algebraic Riccati equation

$$A^TX + XA + XBB^TX + Q = 0$$

This solution can be computed directly with the Riccati solver `care` and compared to the minimizer returned by `mincx`.

From an LMI optimization standpoint, the problem specified in Equation 4-9 is equivalent to the following linear objective minimization problem:

Minimize Trace(X) subject to

$$\begin{pmatrix} A^TX + XA + Q & XB \\ B^TX & -I \end{pmatrix} < 0.$$

Since Trace(X) is a linear function of the entries of X, this problem falls within the scope of the `mincx` solver and can be numerically solved as follows:

1 Define the LMI constraint of Equation 4-9 by the sequence of commands

```
setlmis([])
X = lmivar(1,[3 1]) % variable X, full symmetric

lmiterm([1 1 1 X],1,a,'s')
lmiterm([1 1 1 0],q)
```

```
lmiterm([1 2 2 0],-1)
lmiterm([1 2 1 X],b',1)

LMIs = getlmis
```

2 Write the objective Trace(X) as c^Tx where x is the vector of free entries of X. Since c should select the diagonal entries of X, it is obtained as the decision vector corresponding to $X = I$, that is,

```
c = mat2dec(LMIs,eye(3))
```

Note that the function `defcx` provides a more systematic way of specifying such objectives (see "Specifying c^Tx Objectives for mincx" on page 4-38 for details).

3 Call `mincx` to compute the minimizer `xopt` and the global minimum `copt = c'*xopt` of the objective:

```
options = [1e-5,0,0,0,0]
[copt,xopt] = mincx(LMIs,c,options)
```

Here `1e-5` specifies the desired relative accuracy on `copt`.

The following trace of the iterative optimization performed by `mincx` appears on the screen:

```
Solver for linear objective minimization under LMI constraints
Iterations  :  Best objective value so far
```

1		
2	-8.511476	
3	-13.063640	
***	new lower bound:	-34.023978
4	-15.768450	
***	new lower bound:	-25.005604
5	-17.123012	
***	new lower bound:	-21.306781
6	-17.882558	
***	new lower bound:	-19.819471
7	-18.339853	

***	new lower bound:	-19.189417
8	-18.552558	
***	new lower bound:	-18.919668
9	-18.646811	
***	new lower bound:	-18.803708
10	-18.687324	
***	new lower bound:	-18.753903
11	-18.705715	
***	new lower bound:	-18.732574
12	-18.712175	
***	new lower bound:	-18.723491
13	-18.714880	
***	new lower bound:	-18.719624
14	-18.716094	
***	new lower bound:	-18.717986
15	-18.716509	
***	new lower bound:	-18.717297
16	-18.716695	
***	new lower bound:	-18.716873

```
Result: feasible solution of required accuracy
 best objective value:  —18.716695
 guaranteed relative accuracy:  9.50e—06
 f-radius saturation: 0.000% of R = 1.00e+09
```

The iteration number and the best value of $c^T x$ at the current iteration appear in the left and right columns, respectively. Note that no value is displayed at the first iteration, which means that a feasible x satisfying the constraint (Equation 4-10) was found only at the second iteration. Lower bounds on the global minimum of $c^T x$ are sometimes detected as the optimization progresses. These lower bounds are reported by the message

```
*** new lower bound: xxx
```

Upon termination, `mincx` reports that the global minimum for the objective Trace(X) is –18.716695 with relative accuracy of at least 9.5×10^{-6}. This is the value `copt` returned by `mincx`.

4 `mincx` also returns the optimizing vector of decision variables `xopt`. The corresponding optimal value of the matrix variable X is given by

```
Xopt = dec2mat(LMIs,xopt,X)
```

which returns

$$X_{opt} = \begin{pmatrix} -6.3542 & -5.8895 & 2.2046 \\ -5.8895 & -6.2855 & 2.2201 \\ 2.2046 & 2.2201 & -6.0771 \end{pmatrix}.$$

This result can be compared with the stabilizing Riccati solution computed by `care`:

```
Xst = care(a,b,q,-1)
norm(Xopt-Xst)

ans =
 6.5390e-05
```

Related Examples

- "Conversion Between Decision and Matrix Variables" on page 4-28

Conversion Between Decision and Matrix Variables

While LMIs are specified in terms of their matrix variables $X_1, \ldots, X_K$, the LMI solvers optimize the vector x of free scalar entries of these matrices, called the decision variables. The two functions `mat2dec` and `dec2mat` perform the conversion between these two descriptions of the problem variables.

Consider an LMI system with three matrix variables X_1, X_2, X_3. Given particular values `X1`, `X2`, `X3` of these variables, the corresponding value `xdec` of the vector of decision variables is returned by `mat2dec`:

```
xdec = mat2dec(LMISYS,X1,X2,X3)
```

An error is issued if the number of arguments following `LMISYS` differs from the number of matrix variables in the problem (see `matnbr`).

Conversely, given a value `xdec` of the vector of decision variables, the corresponding value of the k-th matrix is given by `dec2mat`. For instance, the value `X2` of the second matrix variable is extracted from `xdec` by

```
X2 = dec2mat(LMISYS,xdec,2)
```

The last argument indicates that the second matrix variable is requested. It could be set to the matrix variable identifier returned by `lmivar`.

The total numbers of matrix variables and decision variables are returned by `matnbr` and `decnbr`, respectively. In addition, the function `decinfo` provides precise information about the mapping between decision variables and matrix variable entries.

Related Examples

- “Validating Results” on page 4-29

Validating Results

The LMI Lab offers two functions to analyze and validate the results of an LMI optimization. The function `evallmi` evaluates all variable terms in an LMI system for a given value of the vector of decision variables, for instance, the feasible or optimal vector returned by the LMI solvers. Once this evaluation is performed, the left and right sides of a particular LMI are returned by `showlmi`.

In the LMI problem considered in "Minimize Linear Objectives under LMI Constraints" on page 4-24, you can verify that the minimizer `xopt` returned by `mincx` satisfies the LMI constraint (Equation 4-10) as follows:

```
evlmi = evallmi(LMIs,xopt)
[lhs,rhs] = showlmi(evlmi,1)
```

The first command evaluates the system for the value `xopt` of the decision variables, and the second command returns the left and right sides of the first (and only) LMI. The negative definiteness of this LMI is checked by

```
eig(lhs-rhs)

ans =
 -2.0387e-04
 -3.9333e-05
 -1.8917e-07
 -4.6680e+01
```

Related Examples

- "Modify a System of LMIs" on page 4-30

Modify a System of LMIs

Once specified, a system of LMIs can be modified in several ways with the functions `dellmi`, `delmvar`, and `setmvar`.

Deleting an LMI

The first possibility is to remove an entire LMI from the system with `dellmi`. For instance, suppose that the LMI system of "Specifying LMI System" on page 4-9 is described in `LMISYS` and that we want to remove the positivity constraint on X. This is done by

```
NEWSYS = dellmi(LMISYS,2)
```

where the second argument specifies deletion of the second LMI. The resulting system of two LMIs is returned in `NEWSYS`.

The LMI identifiers (*initial* ranking of the LMI in the LMI system) are not altered by deletions. As a result, the last LMI

$S > I$

remains known as the third LMI even though it now ranks second in the modified system. To avoid confusion, it is safer to refer to LMIs via the identifiers returned by `newlmi`. If `BRL`, `Xpos`, and `Slmi` are the identifiers attached to the three LMIs, Equation 4-6–Equation 4-8, `Slmi` keeps pointing to $S > I$ even after deleting the second LMI by

```
NEWSYS = dellmi(LMISYS,Xpos)
```

Deleting a Matrix Variable

Another way of modifying an LMI system is to delete a matrix variable, that is, to remove all variable terms involving this matrix variable. This operation is performed by `delmvar`. For instance, consider the LMI

$A^T X + XA + BW + W^T B^T + I < 0$

with variables $X = X^T \in R^{4\times 4}$ and $W \in R^{2\times 4}$. This LMI is defined by

```
setlmis([])
```

```
X = lmivar(1,[4 1])  % X
W = lmivar(2,[2 4])  % W

lmiterm([1 1 1 X],1,A,'s')
lmiterm([1 1 1 W],B,1,'s')
lmiterm([1 1 1 0],1)

LMISYS = getlmis
```

To delete the variable `W`, type the command

```
NEWSYS = delmvar(LMISYS,W)
```

The resulting `NEWSYS` now describes the Lyapunov inequality

$$A^T X + XA + I < 0$$

Note that `delmvar` automatically removes all LMIs that depended only on the deleted matrix variable.

The matrix variable identifiers are not affected by deletions and continue to point to the same matrix variable. For subsequent manipulations, it is therefore advisable to refer to the remaining variables through their identifier. Finally, note that deleting a matrix variable is equivalent to setting it to the zero matrix of the same dimensions with `setmvar`.

Instantiating a Matrix Variable

The function `setmvar` is used to set a matrix variable to some given value. As a result, this variable is removed from the problem and all terms involving it become constant terms. This is useful, for instance, to fix`setmvar` some variables and optimize with respect to the remaining ones.

Consider again “Specifying LMI System” on page 4-9 and suppose we want to know if the peak gain of G itself is less than one, that is, if

$$\|G\|^{\infty} < 1$$

This amounts to setting the scaling matrix D (or equivalently, $S = D^{\mathrm{T}}D$) to a multiple of the identity matrix. Keeping in mind the constraint $S > I$, a legitimate choice is S = 2-ßψ-I. To set S to this value, enter

```
NEWSYS = setmvar(LMISYS,S,2)
```

The second argument is the variable identifier `S`, and the third argument is the value to which S should be set. Here the value 2 is shorthand for 2-by-I. The resulting system `NEWSYS` reads

$$\begin{pmatrix} A^T X + XA + 2C^{TC} & XB \\ B^T X & -2I \end{pmatrix} < 0$$
$$X > 0$$
$$2I > I.$$

Note that the last LMI is now free of variable and trivially satisfied. It could, therefore, be deleted by

```
NEWSYS = dellmi(NEWSYS,3)
```

or

```
NEWSYS = dellmi(NEWSYS,Slmi)
```

if `Slmi` is the identifier returned by `newlmi`.

Related Examples

- "Advanced LMI Techniques" on page 4-33

Advanced LMI Techniques

This last section gives a few hints for making the most out of the LMI Lab. It is directed toward users who are comfortable with the basics, as described in "Tools for Specifying and Solving LMIs" on page 4-2.

Structured Matrix Variables

Fairly complex matrix variable structures and interdependencies can be specified with `lmivar`. Recall that the symmetric block-diagonal or rectangular structures are covered by Types 1 and 2 of `lmivar` provided that the matrix variables are independent. To describe more complex structures or correlations between variables, you must use Type 3 and specify each entry of the matrix variables directly in terms of the free scalar variables of the problem (the so-called decision variables).

With Type 3, each entry is specified as either 0 or $\pm x_n$ where x_n is the n-th decision variable. The following examples illustrate how to specify nontrivial matrix variable structures with `lmivar`. First, consider the case of uncorrelated matrix variables.

Example: Specifying Matrix Variable Structures

Suppose that the problem variables include a 3-by-3 symmetric matrix X and a 3-by-3 symmetric Toeplitz matrix:

$$Y = \begin{pmatrix} y_1 & y_2 & y_3 \\ y_2 & y_1 & y_2 \\ y_3 & y_2 & y_1 \end{pmatrix}.$$

The variable Y has three independent entries, hence involves three decision variables. Since Y is independent of X, these decision variables should be labeled $n + 1$, $n + 2$, $n + 3$ where n is the number of decision variables involved in X. To retrieve this number, define the variable X (Type 1) by

```
setlmis([])
[X,n] = lmivar(1,[3 1])
```

The second output argument `n` gives the total number of decision variables used so far (here `n = 6`). Given this number, Y can be defined by

```
Y = lmivar(3,n+[1 2 3;2 1 2;3 2 1])
```

or equivalently by

```
Y = lmivar(3,toeplitz(n+[1 2 3]))
```

where `toeplitz` is a standard MATLAB function. For verification purposes, we can visualize the decision variable distributions in X and Y with `decinfo`:

```
lmis = getlmis
decinfo(lmis,X)

ans =
 1 2 4
 2 3 5
 4 5 6

decinfo(lmis,Y)

ans =
 7 8 9
 8 7 8
 9 8 7
```

The next example is a problem with interdependent matrix variables.

Example: Specifying Interdependent Matrix Variables

Consider three matrix variables X, Y, Z with structure

$$X = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}, \quad Y = \begin{pmatrix} z & 0 \\ 0 & t \end{pmatrix}, \quad Z = \begin{pmatrix} 0 & -x \\ -t & 0 \end{pmatrix}$$

where x, y, z, t are independent scalar variables. To specify such a triple, first define the two independent variables X and Y (both of Type 1) as follows:

```
setlmis([])
[X,n,sX] = lmivar(1,[1 0;1 0])
[Y,n,sY] = lmivar(1,[1 0;1 0])
```

The third output of `lmivar` gives the entry-wise dependence of X and Y on the decision variables $(x_1, x_2, x_3, x_4) := (x, y, z, t)$:

```
sX =
```

```
 1   0
 0   2

sY =
 3   0
 0   4
```

Using Type 3 of `lmivar`, you can now specify the structure of `Z` in terms of the decision variables $x_1 = x$ and $x_4 = t$:

```
[Z,n,sZ] = lmivar(3,[0 -sX(1,1);-sY(2,2) 0])
```

Since `sX(1,1)` refers to x_1 while `sY(2,2)` refers to x_4, this defines the variable

$$Z = \begin{pmatrix} 0 & -x_1 \\ -x_4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -x \\ -t & 0 \end{pmatrix}$$

as confirmed by checking its entry-wise dependence on the decision variables:

```
sZ =
 0   -1
 -4   0
```

Complex-Valued LMIs

The LMI solvers are written for real-valued matrices and cannot directly handle LMI problems involving complex-valued matrices. However, complex-valued LMIs can be turned into real-valued LMIs by observing that a complex Hermitian matrix $L(x)$ satisfies

$L(x) < 0$

if and only if

$$\begin{pmatrix} \mathrm{Re}\left(L(x)\right) & \mathrm{Im}\left(L(x)\right) \\ -\mathrm{Im}\left(L(x)\right) & \mathrm{Re}\left(L(x)\right) \end{pmatrix} < 0.$$

This suggests the following systematic procedure for turning complex LMIs into real ones:

- Decompose every complex matrix variable X as

  $X = X_1 + jX_2$

  where X_1 and X_2 are real
- Decompose every complex matrix coefficient A as

  $A = A_1 + jA_2$

  where A_1 and A_2 are real
- Carry out all complex matrix products. This yields affine expressions in X_1, X_2 for the real and imaginary parts of each LMI, and an equivalent real-valued LMI is readily derived from the above observation.

For LMIs without outer factor, a streamlined version of this procedure consists of replacing any occurrence of the matrix variable $X = X_1 + jX_2$ by

$$\begin{pmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{pmatrix}$$

and any fixed matrix $A = A_1 + jA_2$, including real ones, by

$$\begin{pmatrix} A_1 & A_2 \\ -A_2 & A_1 \end{pmatrix}.$$

For instance, the real counterpart of the LMI system

$M^H X M < X, \quad X = X^H > I$

reads (given the decompositions $M = M_1 + jM_2$ and $X = X_1 + jX_2$ with Mj, Xj real):

$$\begin{pmatrix} M_1 & M_2 \\ -M_2 & M_1 \end{pmatrix}^T \begin{pmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{pmatrix} \begin{pmatrix} M_1 & M_2 \\ -M_2 & M_1 \end{pmatrix} < \begin{pmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{pmatrix}$$
$$\begin{pmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{pmatrix} < I.$$

Note that $X = X^H$ in turn requires that $X_1 = X_1^H$ and $X_2 + X_2^T = 0$. Consequently, X_1 and X_2 should be declared as symmetric and skew- symmetric matrix variables, respectively.

Assuming, for instance, that $M \in \mathbf{C}^{5\times 5}$, the LMI system (Equation 4-11) would be specified as follows:

```
M1=real(M), M2=imag(M)
bigM=[M1 M2;-M2 M1]
setlmis([])

% declare bigX=[X1 X2;-X2 X1] with X1=X1' and X2+X2'=0:

[X1,n1,sX1] = lmivar(1,[5 1])
[X2,n2,sX2] = lmivar(3,skewdec(5,n1))
bigX = lmivar(3,[sX1 sX2;-sX2 sX1])

% describe the real counterpart of (1.12):

lmiterm([1 1 1 0],1)
lmiterm([-1 1 1 bigX],1,1)
lmiterm([2 1 1 bigX],bigM',bigM)
lmiterm([-2 1 1 bigX],1,1)

lmis = getlmis
```

Note the three-step declaration of the structured matrix variable `bigX`,

$$bigX = \begin{pmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{pmatrix}.$$

1. Specify X_1 as a (real) symmetric matrix variable and save its structure description `sX1` as well as the number `n1` of decision variables used in X_1.
2. Specify X_2 as a skew-symmetric matrix variable using Type 3 of `lmivar` and the utility `skewdec`. The command `skewdec(5,n1)` creates a 5-by–5 skew-symmetric structure depending on the decision variables `n1 + 1`, `n1 + 2`,...
3. Define the structure of `bigX` in terms of the structures `sX1` and `sX2` of X_1 and X_2.

See “Structured Matrix Variables” on page 4-33 for more details on such structure manipulations.

Specifying $c^T x$ Objectives for mincx

The LMI solver `mincx` minimizes linear objectives of the form $c^T x$ where x is the vector of decision variables. In most control problems, however, such objectives are expressed in terms of the matrix variables rather than of x. Examples include Trace(X) where X is a symmetric matrix variable, or $u^T X u$ where u is a given vector.

The function `defcx` facilitates the derivation of the c vector when the objective is an affine function of the *matrix variables*. For the sake of illustration, consider the linear objective

$$\mathrm{Trace}(X) + x_0^T P x_0$$

where X and P are two symmetric variables and x_0 is a given vector. If `lmsisys` is the internal representation of the LMI system and if x_0, X, P have been declared by

```
x0 = [1;1]
setlmis([])
X = lmivar(1,[3 0])
P = lmivar(1,[2 1])
 :
 :
lmisys = getlmis
```

the c vector such that $c^T x = \mathrm{Trace}(X) + x_0^T P x_0$ can be computed as follows:

```
n = decnbr(lmisys)
c = zeros(n,1)

for j=1:n,
 [Xj,Pj] = defcx(lmisys,j,X,P)
 c(j) = trace(Xj) + x0'*Pj*x0
end
```

The first command returns the number of decision variables in the problem and the second command dimensions c accordingly. Then the `for` loop performs the following operations:

1. Evaluate the matrix variables X and P when all entries of the decision vector x are set to zero except xj: = 1. This operation is performed by the function `defcx`. Apart

from `lmisys` and `j`, the inputs of `defcx` are the identifiers `X` and `P` of the variables involved in the objective, and the outputs `Xj` and `Pj` are the corresponding values.

2. Evaluate the objective expression for `X:= Xj` and `P:= Pj`. This yields the *j*-th entry of `c` by definition.

In our example the result is

```
c =
 3
 1
 2
 1
```

Other objectives are handled similarly by editing the following generic skeleton:

```
n = decnbr( LMI system )
c = zeros(n,1)
for j=1:n,
 [ matrix values ] = defcx( LMI system,j,
matrix identifiers)
 c(j) = objective(matrix values)
end
```

Feasibility Radius

When solving LMI problems with `feasp`, `mincx`, or `gevp`, it is possible to constrain the solution x to lie in the ball

$$x^T x < R^2$$

where $R > 0$ is called the *feasibility radius*. This specifies a maximum (Euclidean norm) magnitude for x and avoids getting solutions of very large norm. This may also speed up computations and improve numerical stability. Finally, the feasibility radius bound regularizes problems with redundant variable sets. In rough terms, the set of scalar variables is redundant when an equivalent problem could be formulated with a smaller number of variables.

The feasibility radius R is set by the third entry of the options vector of the LMI solvers. Its default value is $R = 109$. Setting R to a negative value means "no rigid bound," in which case the feasibility radius is increased during the optimization if necessary. This "flexible bound" mode may yield solutions of large norms.

Well-Posedness Issues

The LMI solvers used in the LMI Lab are based on interior-point optimization techniques. To compute feasible solutions, such techniques require that the system of LMI constraints be strictly feasible, that is, the feasible set has a nonempty interior. As a result, these solvers may encounter difficulty when the LMI constraints are feasible but not *strictly feasible*, that is, when the LMI

$L(x) \leq 0$

has solutions while

$L(x) < 0$

has no solution.

For feasibility problems, this difficulty is automatically circumvented by `feasp`, which reformulates the problem:

Find x such that

$L(x) \leq 0$

as:

Minimize t subject to

$Lx < t \times I$.

In this modified problem, the LMI constraint is always strictly feasible in x, t and the original LMI Equation 4-12 is feasible if and only if the global minimum $t_{\min}$ of Equation 4-12 satisfies

$t_{\min} \leq 0$

For feasible but not strictly feasible problems, however, the computational effort is typically higher as `feasp` strives to approach the global optimum $t_{\min} = 0$ to a high accuracy.

For the LMI problems addressed by `mincx` and `gevp`, nonstrict feasibility generally causes the solvers to fail and to return an “infeasibility” diagnosis. Although there is

no universal remedy for this difficulty, it is sometimes possible to eliminate underlying algebraic constraints to obtain a strictly feasible problem with fewer variables.

Another issue has to do with homogeneous feasibility problems such as

$A^TP + PA < 0, P > 0$

While this problem is technically well-posed, the LMI optimization is likely to produce solutions close to zero (the trivial solution of the nonstrict problem). To compute a nontrivial Lyapunov matrix and easily differentiate between feasibility and infeasibility, replace the constraint $P > 0$-by-$P > \alpha I$ with $\alpha > 0$. Note that this does not alter the problem due to its homogeneous nature.

Semi-Definite B(x) in gevp Problems

Consider the generalized eigenvalue minimization problem

Minimize λ subject to

$A(x) < \lambda B(x), B(x) > 0, C(x) < 0.$

Technically, the positivity of $B(x)$ for some $x \in R^n$ is required for the well-posedness of the problem and the applicability of polynomial-time interior-point methods. Hence problems where

$$B(x) = \begin{pmatrix} B_1(x) & 0 \\ 0 & 0 \end{pmatrix}$$

with $B_1(x) > 0$ strictly feasible, cannot be directly solved with `gevp`. A simple remedy consists of replacing the constraints

$A(x) < B(x), B(x) > 0$

by

$$A(x) < \begin{pmatrix} Y & 0 \\ 0 & 0 \end{pmatrix}, \quad Y < \lambda B_1(x), \quad B_1(x) > 0$$

where Y is an additional symmetric variable of proper dimensions. The resulting problem is equivalent to Equation 4-14 and can be solved directly with `gevp`.

Efficiency and Complexity Issues

As explained in "Tools for Specifying and Solving LMIs" on page 4-2, the term-oriented description of LMIs used in the LMI Lab typically leads to higher efficiency than the canonical representation

$$A_0 + x_1A_1 + \ldots + x_NA_N < 0.$$

This is no longer true, however, when the number of variable terms is nearly equal to or greater than the number N of decision variables in the problem. If your LMI problem has few free scalar variables but many terms in each LMI, it is therefore preferable to rewrite it as Equation 4-15 and to specify it in this form. Each scalar variable x_j is then declared independently and the LMI terms are of the form x_jA_j.

If M denotes the total row size of the LMI system and N the total number of scalar decision variables, the flop count per iteration for the `feasp` and `mincx` solvers is proportional to

- N^3 when the least-squares problem is solved via. Cholesly factorization of the Hessian matrix (default) [2]
- M-by-N^2 when numerical instabilities warrant the use of QR factorization instead

While the theory guarantees a worst-case iteration count proportional to M, the number of iterations actually performed grows slowly with M in most problems. Finally, while `feasp` and `mincx` are comparable in complexity, `gevp` typically demands more computational effort. Make sure that your LMI problem cannot be solved with `mincx` before using `gevp`.

Solving $M + P^TXQ + Q^TX^TP < 0$

In many output-feedback synthesis problems, the design can be performed in two steps:

1. Compute a closed-loop Lyapunov function via LMI optimization.
2. Given this Lyapunov function, derive the controller state-space matrices by solving an LMI of the form

   $$M + P^TXQ + Q^TX^TP < 0$$

   where M, P, Q are given matrices and X is an unstructured m-by-n matrix variable.

It turns out that a particular solution X_c of Equation 4-16 can be computed via simple linear algebra manipulations [1]. Typically, X_c corresponds to the center of the ellipsoid of matrices defined by Equation 4-16.

The function `basiclmi` returns the "explicit" solution X_c:

```
Xc = basiclmi(M,P,Q)
```

Since this central solution sometimes has large norm, `basiclmi` also offers the option of computing an approximate least-norm solution of Equation 4-16. This is done by

```
X = basiclmi(M,P,Q,'Xmin')
```

and involves LMI optimization to minimize $\|X\|$.

More About

- "Tools for Specifying and Solving LMIs" on page 4-2

Bibliography

[1] Gahinet, P., and P. Apkarian, "A Linear Matrix Inequality Approach to H^∞ Control," *Int. J. Robust and Nonlinear Contr.*, 4 (1994), pp. 421-448.

[2] Nemirovski, A., and P. Gahinet, "The Projective Method for Solving Linear Matrix Inequalities," *Proc. Amer. Contr. Conf.*, 1994, pp. 840-844.

[3] Nesterov, Yu, and A. Nemirovski, *Interior Point Polynomial Methods in Convex Programming: Theory and Applications*, SIAM Books, Philadelphia, 1994.

[4] Shamma, J.S., "Robustness Analysis for Time-Varying Systems," *Proc. Conf. Dec. Contr.*, 1992, pp. 3163-3168.

5

Analyzing Uncertainty Effects in Simulink

Analyzing Uncertainty in Simulink

Robust Control Toolbox software provides tools to model uncertainty in Simulink. Using these tools, you can analyze how the uncertainty impacts the time- and frequency-domain behavior of the Simulink model.

The `Uncertain State Space` block, included in the Robust Control Toolbox block library, is a convenient way to incorporate uncertainty information in a Simulink model. For more information, see "Specify Uncertainty Using Uncertain State Space Blocks" on page 5-4. Using this block, you can perform the following types of analysis:

- Vary the uncertainty and see how it affects the time responses (Monte Carlo analysis). See "Simulate Uncertainty Effects" on page 5-7.
- Analyze the effects of uncertainty on the linearized dynamics:
  - If the operating point does not depend on the parameter uncertainty, use `ulinearize` to obtain an uncertain state-space model. You can then use `usample` to sample the uncertain variables and obtain a family of LTI models.
  - If the operating point depends on the parameter uncertainty, use `usample` to sample the uncertainty and then use the Simulink Control Design™ `linearize` command to compute the linearized dynamics for each uncertainty value.

    See "How to Vary Uncertainty Values" on page 5-7 and "Obtain Uncertain State-Space Model from Simulink Model" on page 5-20.
- Compute an *uncertain linearization*, i.e., obtain an uncertain state-space model (`uss` object) that combines the uncertain variables with the linearized dynamics. You can use this model to perform worst-case robustness analysis. See "Obtain Uncertain State-Space Model from Simulink Model" on page 5-20.

If you cannot use `Uncertain State Space` blocks in the Simulink model because you share the model or generate code, you can still compute an uncertain linearization by specifying a block to linearize to an uncertain variable. For example, you can specify a gain block to linearize to an uncertain real parameter (`ureal`). See "Specify Uncertain Linearization for Core or Custom Simulink Blocks" on page 5-21. You can then use the uncertain state-space model to analyze robustness in the linear operating range.

Simulink Blocks for Analyzing Uncertainty

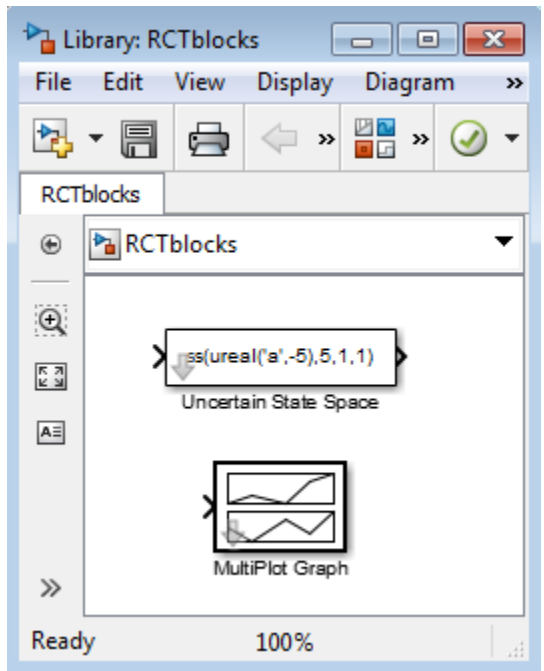
Robust Control Toolbox software provides an `Uncertain State Space` block to model parametric and dynamic uncertainty in Simulink. The block library also contains a

`MultiPlot Graph` block that you use with the `Uncertain State Space` block to plot and visualize Monte Carlo simulation responses.

To open the Robust Control Toolbox block library, type the following command at the MATLAB prompt:

```
RCTblocks
```

The block library opens, as shown in the following figure.



Alternatively, in a Simulink model window, click  to launch to Library Browser. In the Library Browser, select **Robust Control Toolbox**.

See Also

`linearize` | `ulinearize` | `Uncertain State Space`

Related Examples

- “Obtain Uncertain State-Space Model from Simulink Model” on page 5-20

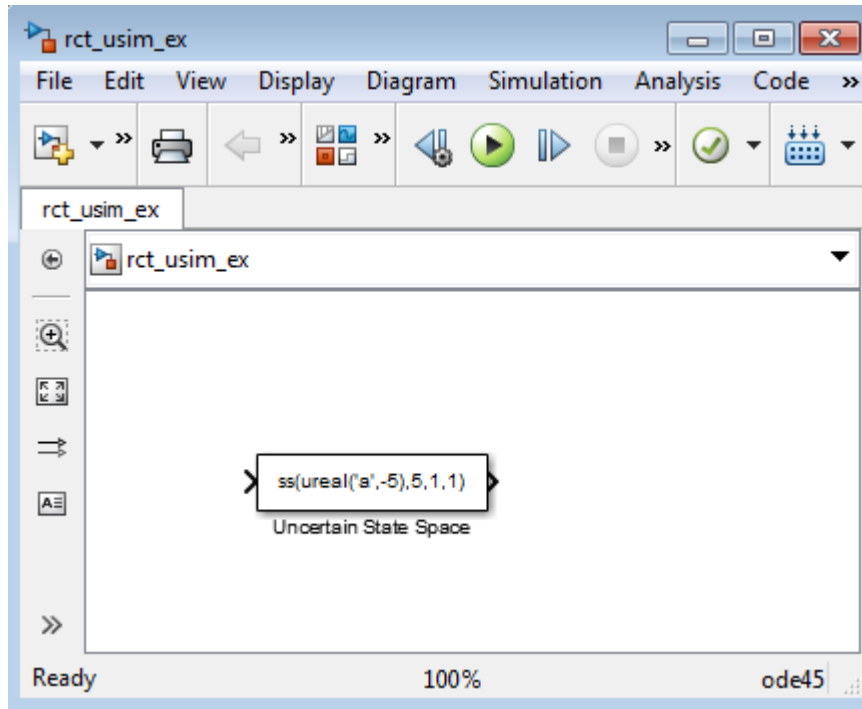
Specify Uncertainty Using Uncertain State Space Blocks

How to Specify Uncertainty in Uncertain State Space Blocks

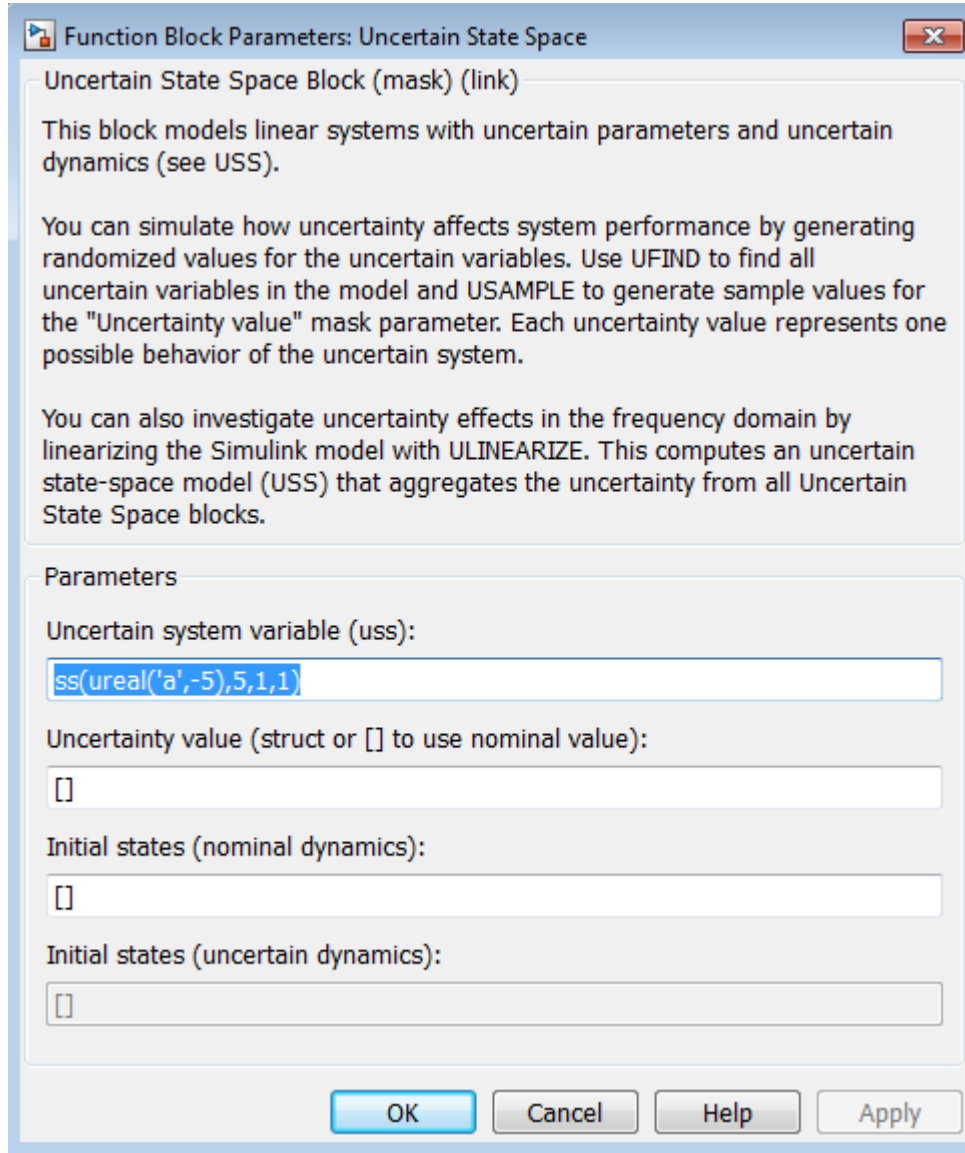
Specifying uncertainty in the `Uncertain State Space` block makes the uncertainty a part of the Simulink model and affects both simulation and linearization. Use this approach to vary the uncertainty and analyze the effects on simulation or linearization.

To specify uncertainty in the `Uncertain State Space` block:

1. Drag and drop an `Uncertain State Space` block from the Robust Control Toolbox block library into a Simulink model. For more information on how to open the block library, see "Simulink Blocks for Analyzing Uncertainty" on page 5-2.



2. In the Simulink model, double-click the `Uncertain State Space` block to open the Function Block Parameters: Uncertain State Space dialog box, as shown in the following figure.



3 Specify an uncertain state-space model in the **Uncertain system variable (uss)** field. The model must be an `uss` object or any other model that can be converted to

`uss`, such as `umat`, `ureal` and `ultidyn`. The model depends on a set of uncertain variables (`ureal` or `ultidyn`) and you can specify the model as one of the following:

- Function or expression that evaluates to an `uss` model. For example, `ss(ureal('a',-5),5,1,1)`.
- Variable, defined in the MATLAB workspace. For example, `unc_sys`, where `unc_sys` is defined as `ss(ureal('a',-5),5,1,1)` in the workspace.

4 Specify values for the uncertain variables that the uncertain state-space model you specify in step 3 uses. For example, if you specify the uncertain system as `ureal('g',2)*tf(1,[ureal('tau'),1])`, then you must specify values for the uncertain variables `g` and `tau`. To do so, enter a structure with fields `g` and `tau` in the **Uncertainty value (struct or [] to use nominal value)** field. You can also enter `[]` to use the nominal values of the uncertain parameters `g` and `tau`.

Tip: You can also use this field to vary the uncertainty values for performing Monte Carlo simulation. For more information, see "Simulate Uncertainty Effects" on page 5-7.

5 (Optional) Specify the initial states of the nominal and uncertain dynamics in the **Initial states (nominal dynamics)** and **Initial states (uncertain dynamics)** fields, respectively.

For more information on the block parameters, see the `Uncertain State Space` block reference page.

Next Steps

After you specify uncertainty in `Uncertain State Space` blocks, you can perform one of the following:

- Simulate the model using nominal, manually-defined or random values, as described in "Simulate Uncertainty Effects" on page 5-7.
- Perform an uncertain linearization, as described in "Obtain Uncertain State-Space Model from Simulink Model" on page 5-20.

Simulate Uncertainty Effects

How to Simulate Effects of Uncertainty

As described in "Specify Uncertainty Using Uncertain State Space Blocks" on page 5-4, the uncertain state-space model you specify in the `Uncertain State Space` block depends on a set of uncertain variables (`ureal` or `ultidyn` objects.) You can simulate the model using nominal value of these uncertain variables. Additionally, you can sample these uncertain variables and simulate the model for various values in the uncertainty range (Monte Carlo simulation.) For more information, see "How to Vary Uncertainty Values" on page 5-7. You can view and compare the simulation results for various sample values of uncertainty using the `MultiPlot Graph` block.

How to Vary Uncertainty Values

There are two ways to control the uncertainty values using the **Uncertainty value (struct or [] to use nominal value)** field of the `Uncertain State Space` block parameters dialog box:

- For simple models with few uncertain variables or one `Uncertain State Space` block, type the value in the `Uncertain State Space` block itself. For more information, see "Vary Uncertainty Values Using Individual Uncertain State Space Blocks" on page 5-8.
- For complex models with large number of uncertain variables or `Uncertain State Space` blocks, use a single data structure for all uncertain variables referenced by the model. Using this approach, you can collectively control the values of all or a subset of uncertain variables and toggle between nominal and user-defined values from the MATLAB prompt. For more information, see "Vary Uncertainty Values Across Multiple Uncertain State Space Blocks" on page 5-15.

Vary Uncertainty Values Using Individual Uncertain State Space Blocks

This example shows the workflow for varying uncertainty values using individual `Uncertain State Space` blocks in a Simulink model. Use this approach for simple models with few uncertain variables or one `Uncertain State Space` block.

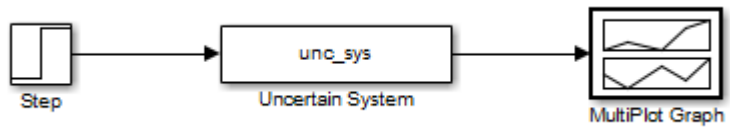
This section uses a simple Simulink model to provide step-by-step instructions for:

- Toggling between nominal, manually defined and randomly-generated uncertainty values associated with the `Uncertain State Space` block.
- Simulating the model's responses for these uncertainty values.

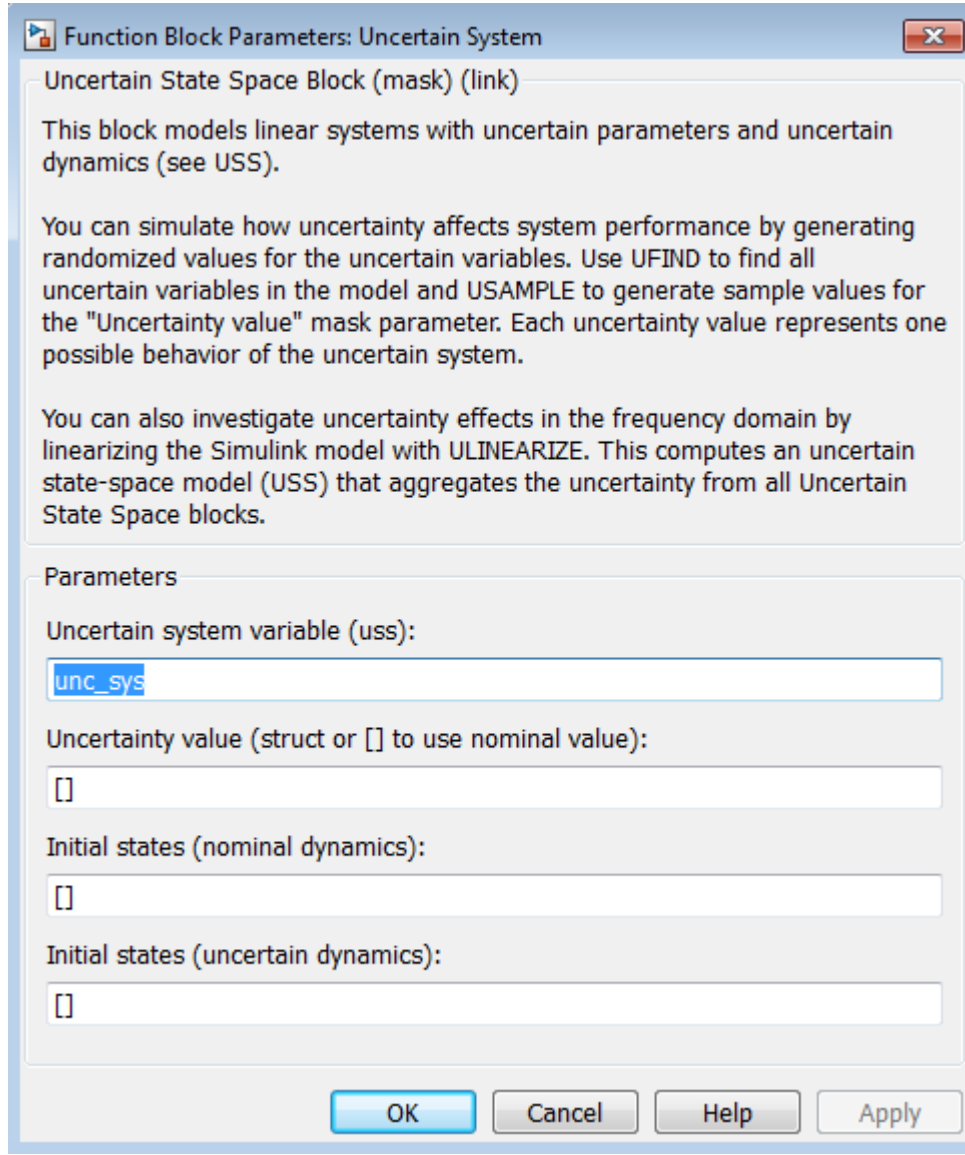
1 Open the Simulink model `rct_sim_ex1`.

```
rct_sim_ex1
```

The model contains an `Uncertain State Space` block called `Uncertain System`, as shown in the following figure.



2 Double-click the `Uncertain System` block to open the Function Block Parameters dialog box.



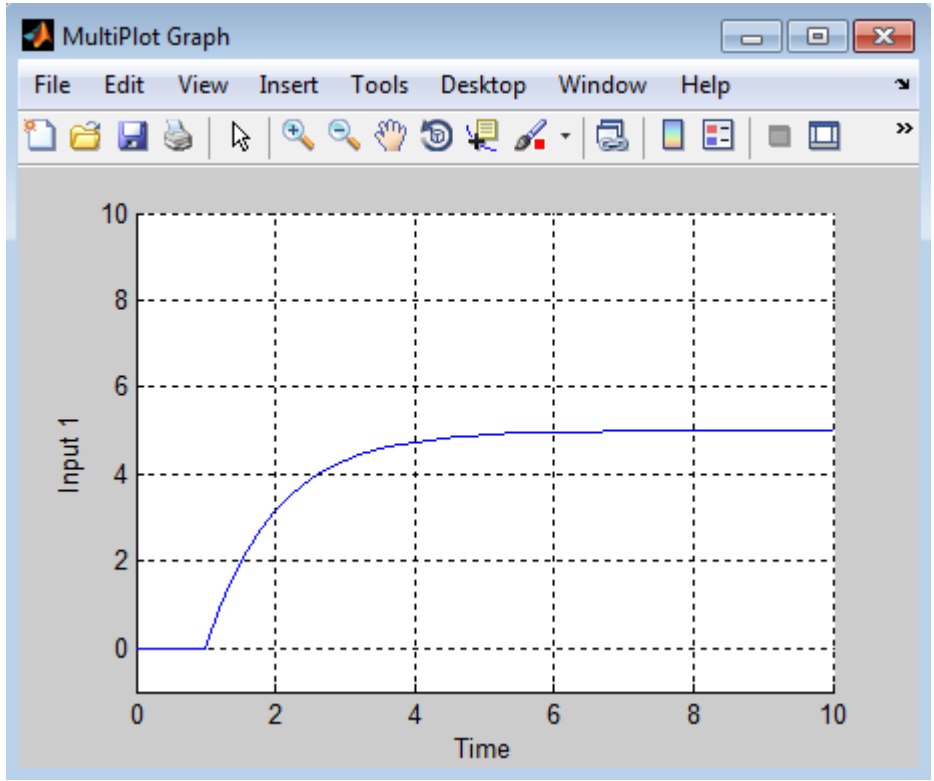
The `Uncertain System` block uses the uncertain system variable `unc_sys`. This variable is defined in the Model workspace as `unc_sys=`

`ss(ureal('a',-1,'Range',[-2 -.5]),1,5,0)*(1+0.1*input_unc)`. The uncertain model depends on a single uncertain variable named `a`. The **Uncertainty value** field specifies to use nominal value of the uncertain variable `a`.

Click **OK** to close the dialog box.

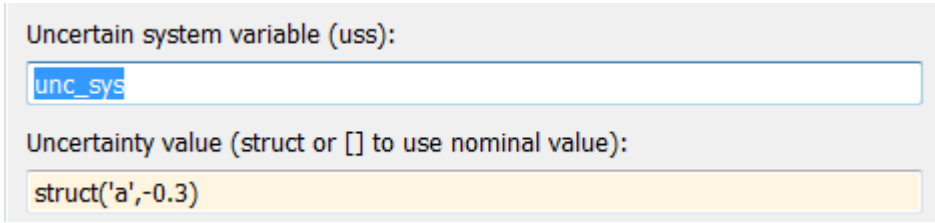
3 Click  to simulate the model.

The software uses the nominal value of `a` during simulation. After the simulation completes, the `MultiPlot Graph` shows the following plot.



4 To simulate the model using a manually defined value of `a`:

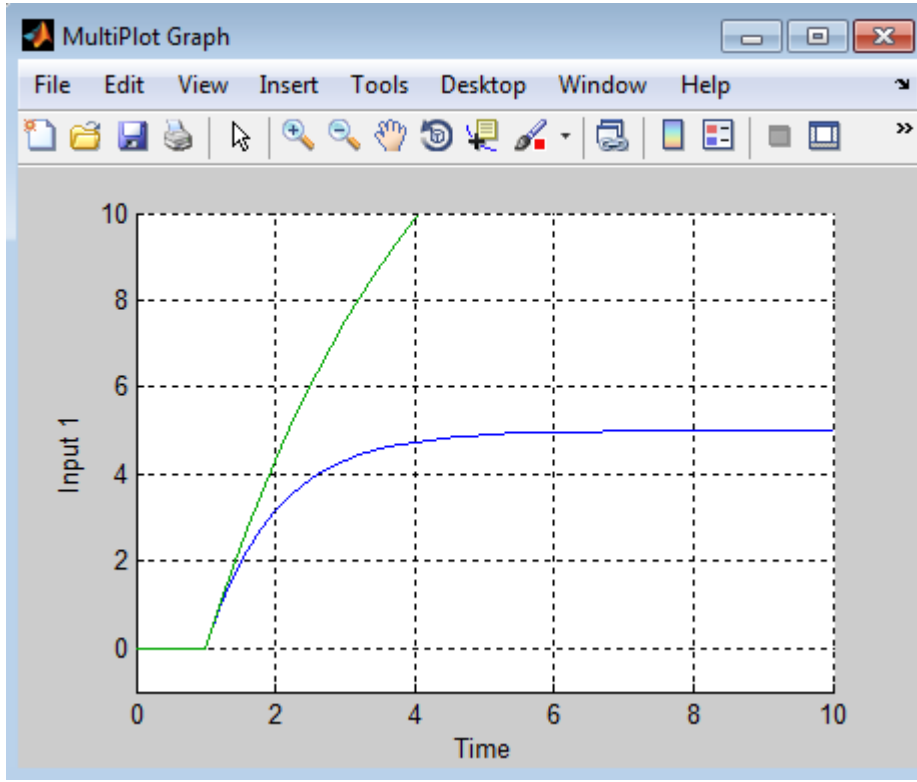
a Double-click the `Uncertain State Space` block, and enter `struct('a',-0.3)` in the **Uncertainty value** field.



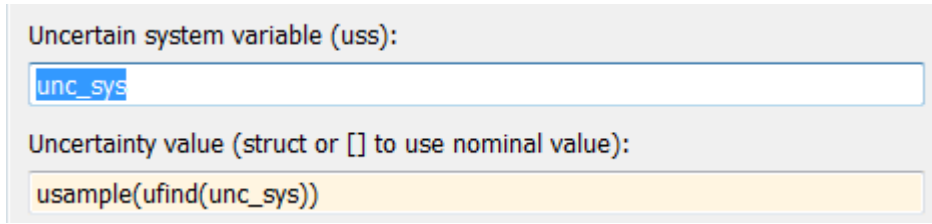
b Click

to simulate the model.

The `MultiPlot Graph` shows the following responses, corresponding to the nominal and manually-defined values of `a`.



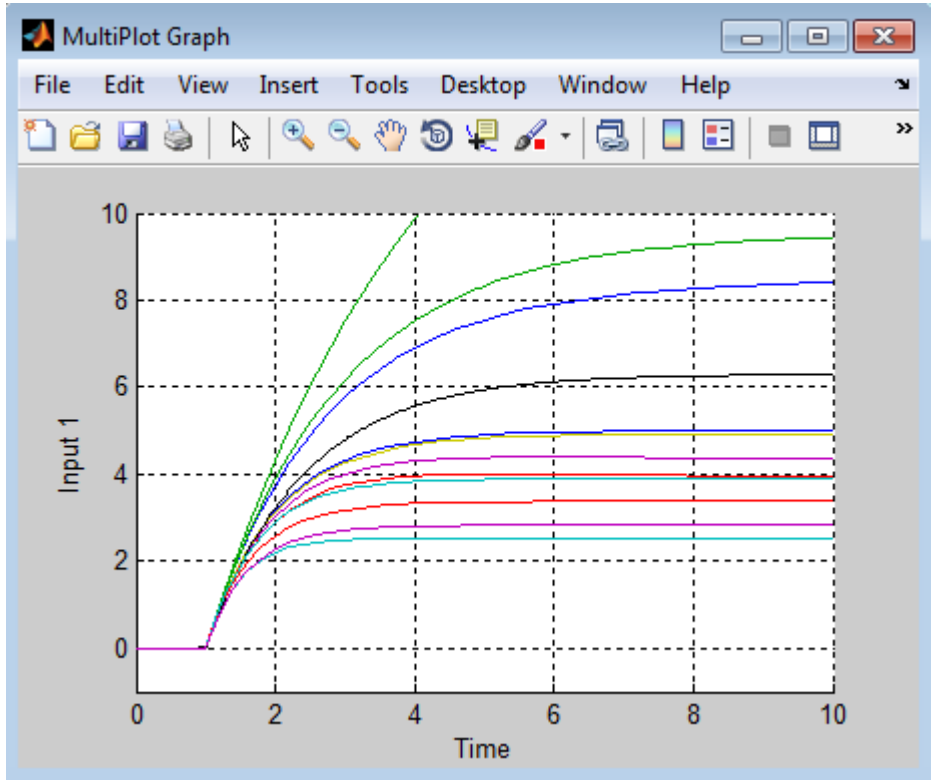
5 Pick a random value of `a` in its uncertainty range. To do so, double-click the `Uncertain State Space` block, and type `usample(ufind(unc_sys))` in the **Uncertainty value** field.



6 Analyze the uncertainty effects by simulating the model ten times using the following commands:

```
for i=1:10;
     sim('rct_sim_ex1',10);
end
```

During simulation, the software samples the uncertain variable `a` in its uncertainty range [-2 -0.5] and shows the simulated response for each sample value. The plots cycle through seven different colors, and the last response appears in red.



Tip: You can clear the plots in the `MultiPlot Graph` block before you run the simulation.

See Also

`MultiPlot Graph` | `Uncertain State Space`

Related Examples

- “Vary Uncertainty Values Across Multiple Uncertain State Space Blocks” on page 5-15

Vary Uncertainty Values Across Multiple Uncertain State Space Blocks

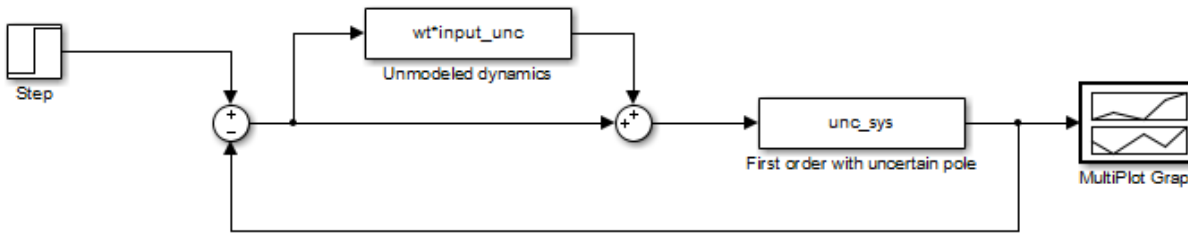
This example shows the workflow for varying uncertainty values across multiple `Uncertain State Space` blocks in a Simulink model. Use this approach for complex models with large number of uncertain variables or `Uncertain State Space` blocks.

This section uses a Simulink model to provide step-by-step instructions for toggling between nominal and user-defined uncertainty values at the MATLAB prompt.

1 Open the Simulink model `rct_sim_ex2`.

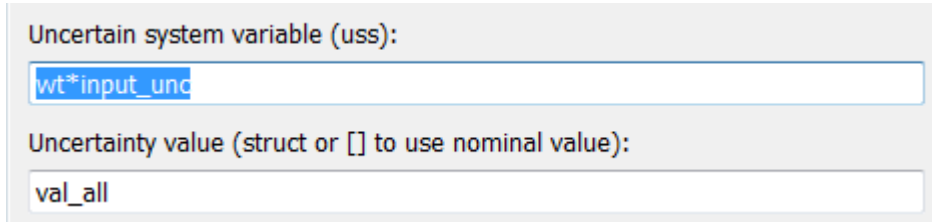
```
rct_sim_ex2
```

The model contains two `Uncertain State Space` blocks, as shown in the following figure.



The `Unmodeled dynamics` and `First order with uncertain pole` blocks depend on the uncertain variables `input_unc` and `a`.

2 Double-click the `Unmodeled dynamics` block to open the block parameters dialog box. The **Uncertainty value** field contains the variable `val_all`. Similarly, the **Uncertainty value** field in the `First order with uncertain pole` block parameters dialog contains the variable `val_all`. You use this variable to vary the uncertain variable values across both the `Uncertain State Space` blocks.

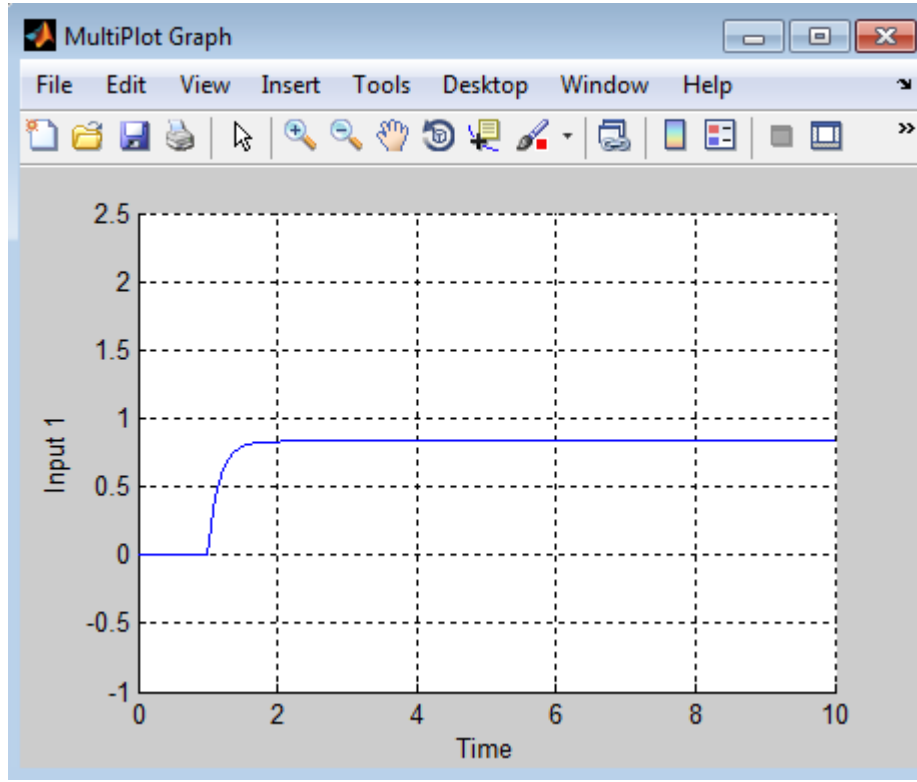


Note When defining `val_all`, you can enter only a subset of uncertain variables referenced by the model in the structure. When you do not specify some uncertain variables, the software uses their nominal value during simulation.

3 At the MATLAB prompt, specify `val_all = [];` and click

to simulate the model.

The software uses the nominal values of the uncertain variables `a` and `input_unc` during simulation. After the simulation completes, the `MultiPlot Graph` block shows the following figure.



4 Generate random samples of uncertainty values:

a Find all `Uncertain State Space` blocks and associated uncertain variables in the model.

```
uvars=ufind('rct_sim_ex2')
```

MATLAB returns the following result:

```
uvars =

          a: [1x1 ureal]
  input_unc: [1x1 ultidyn]
```

The uncertain variables `a` and `input_unc` are `ureal` and `ultidyn` objects, respectively and the structure `uvars` lists them by name.

b Randomly sample the uncertain variables.

```
val_all = usample(uvars)
```

MATLAB returns the following result:

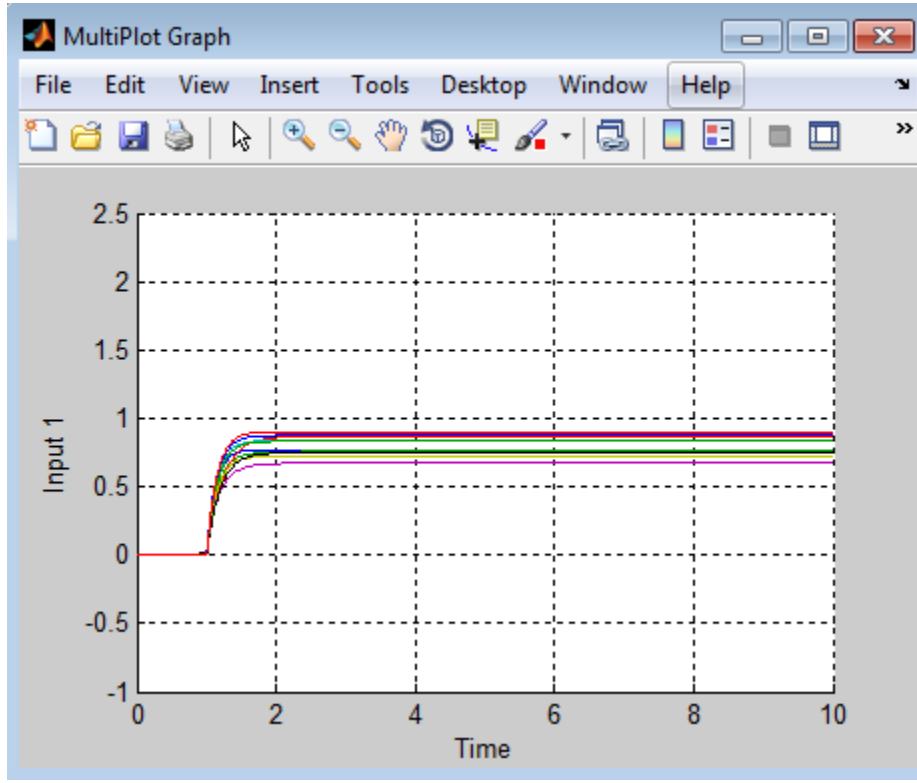
```
val_all =

          a: -1.1167
  input_unc: [1x1 ss]
```

The structure `val_all` contains sample values of the uncertain variables `a` and `input_unc`. The software samples the values within the specified uncertainty ranges for `a` and `input_unc`.

5 Simulate the model for the uncertainty values `val_all`. By repeating the process inside a for-loop, you can assess how uncertainty affects the model responses. For example, perform 10 simulations using random uncertainty values:

```
for i=1:10;
       val_all = usample(uvars)
       sim('rct_sim_ex2',10);
end
```

During each simulation, the software samples values of the uncertain variables `input_unc` and `a` and plots the response for the sampled values. The `MultiPlot Graph` block shows the following responses obtained using random sample values of uncertain variables.


MultiPlot Graph
File
Edit
View
Insert
Tools
Desktop
Window
Help
2.5
2
1.5
1
0.5
0
-0.5
-1
Input 1
0
2
4
6
8
10
Time

Compute Uncertain State-Space Models from Simulink Models

When you have the Simulink Control Design software, you can compute an *uncertain linearization*, i.e., an uncertain state-space model (`uss`) combining the uncertain variables with linearized dynamics. Use the `uss` model to perform linear analysis and robust control design.

You can compute an uncertain linearization in one of the following ways:

- Using the `ulinearize` command, as described in "Obtain Uncertain State-Space Model from Simulink Model" on page 5-20.
- Using the Simulink Control Design `linearize` command, as described in "Specify Uncertain Linearization for Core or Custom Simulink Blocks" on page 5-21.

Obtain Uncertain State-Space Model from Simulink Model

To obtain an uncertain state-space model from a model that contains `Uncertain State Space` blocks, use the following steps:

Note If you do not have `Uncertain State Space` blocks in the model but still want to obtain an uncertain state-space model, see "Specify Uncertain Linearization for Core or Custom Simulink Blocks" on page 5-21.

---

1. (Prerequisite) Create or open the Simulink model.
2. (Prerequisite) In the Simulink model, specify the linearization input and output points using Simulink Control Design `getlinio` or `linio` commands. For more information, see "Specifying Portion of Model to Linearize" in the Simulink Control Design documentation.
3. (Prerequisite) If you have not already done so, specify uncertainty in the Simulink model as described in "Specify Uncertainty Using Uncertain State Space Blocks" on page 5-4.

   Note The software does not evaluate the uncertain variables during linearization. Thus, the value of the uncertainty does not affect the linearization.

   ---

4. Run `ulinearize` to compute an uncertain linearization. This command returns an `uss` model.

Note If you use the Simulink Control Design`linearize` command, the `Uncertain State Space` blocks linearize to their nominal value.

For more information on linearization and how to evaluate the results, see "Linearization Basics" in the Simulink Control Design documentation.

For an example of how to use the Simulink Control Design `linearize` command, see "Linearization of Simulink Models with Uncertainty".

Specify Uncertain Linearization for Core or Custom Simulink Blocks

In some cases, you cannot use `Uncertain State Space` blocks in the Simulink model because you share the model or generate code. You can still account for uncertainty in your linear analysis without specifying uncertainty using `Uncertain State Space` blocks. Robust Control Toolbox lets you specify a core or custom Simulink block to linearize to an uncertain variable. The linearization produces an uncertain state-space `uss` model. The specified uncertainty associates only with the block and does not affect the model simulation. For more information, see "Specify Linear System for Block Linearization Using MATLAB Expression" in the Simulink Control Design documentation.

Note If you have `Uncertain State Space` blocks in the model and want to obtain an uncertain state-space model, see "Obtain Uncertain State-Space Model from Simulink Model" on page 5-20.

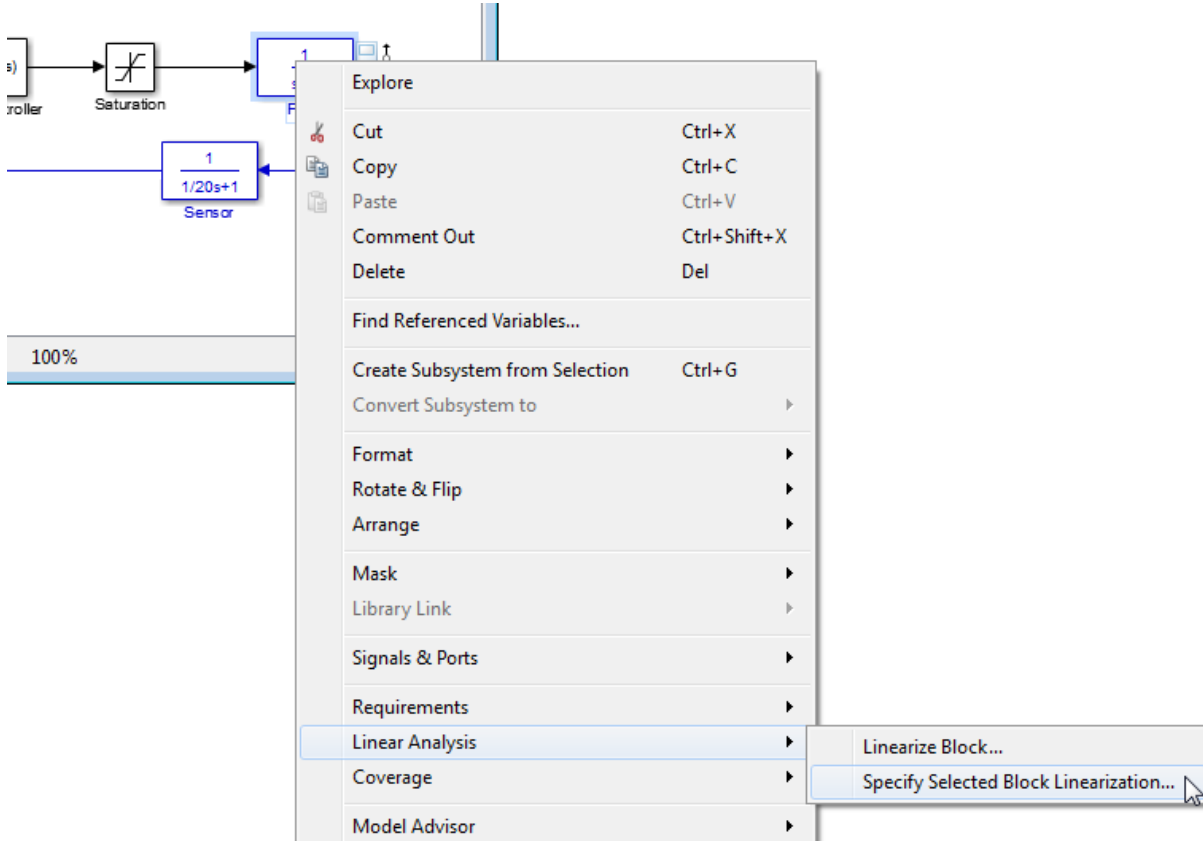
To specify blocks to linearize to uncertain variables and obtain an uncertain state-space model:

1. (Prerequisites) Create or open the Simulink model. Specify linearization input and output points using the Simulink Control Design `getlinio` or `linio` commands.

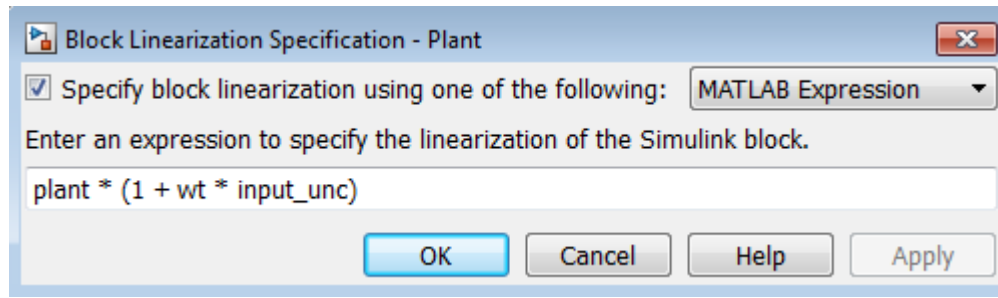
   For this example, you can open the model `rct_ulinearize_builtin`.

2. Specify a block to linearize to an uncertain variable:

   a. Right-click the block and select **Linear Analysis > Specify Selected Block Linearization.**



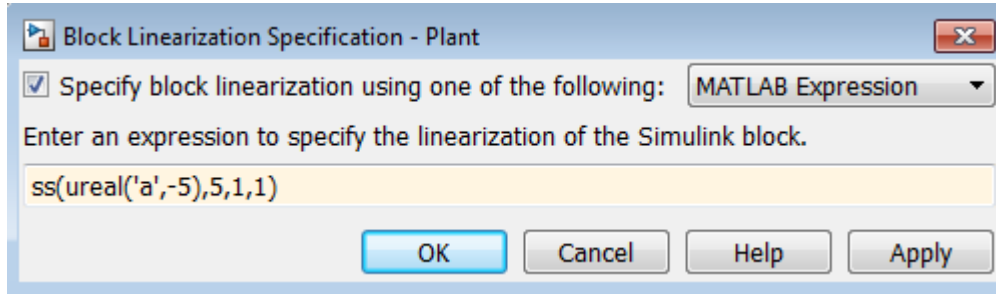
This action opens the Block Linearization Specification dialog box.



b In the Block Linearization Specification dialog box, select the **Specify block linearization using one of the following:** check box. Selecting this check box lets you to specify an uncertain variable for linearization.

This check box defaults to `MATLAB Expression` in the drop-down menu. This option lets you specify the block to linearize to an uncertain variable using a MATLAB expression containing Robust Control Toolbox functions. To learn more about the options, see "Specify Linear System for Block Linearization Using MATLAB Expression" in the Simulink Control Design documentation.

c In the **Enter an expression to specify the linearization of the Simulink block** field, enter an expression, which must evaluate to an uncertain variable or uncertain model, such as `ureal`, `umat`, `ultidyn` or `uss`.



d Click **OK** to save the changes.

Note You can also specify a block to linearize to an uncertain variable at the command line. For an example, see "Linearize Block to Uncertain Model" on page 5-25.

---

3 Run the `linearize` command to compute an uncertain linearization. This command returns an `uss` model.

For more information on linearization and how to validate linearization results, see "Linearization Basics" in the Simulink Control Design documentation.

For an example of how to use the `linearize` command to compute an uncertain linearization, see "Linearization of Simulink Models with Uncertainty".

Using Uncertain Linearization for Analysis or Control Design

After computing an uncertain linearization, you can perform any analysis or design tasks you would perform on any linear model, including:

- Perform robustness analysis. See “Robustness and Worst-Case Analysis”.
- Perform robust control design. See “Robust Tuning”.

See Also

`linearize` | `ulinearize`

Related Examples

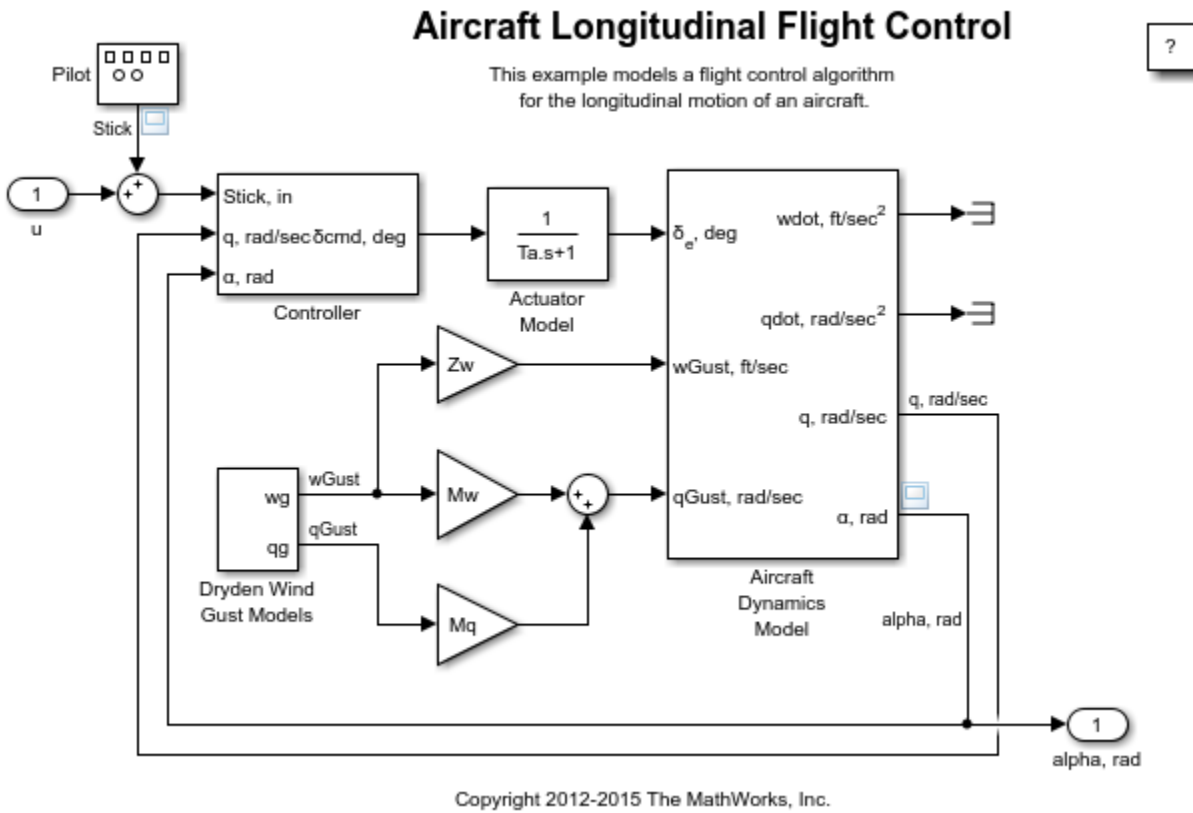
- “Linearization of Simulink Models with Uncertainty”
- “Linearize Block to Uncertain Model” on page 5-25

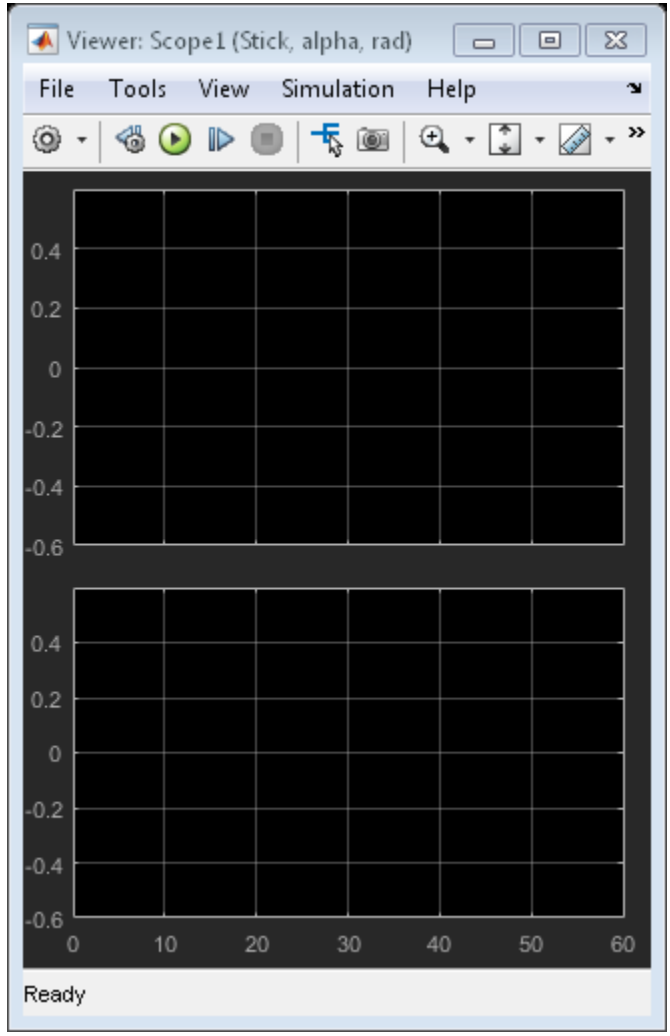
Linearize Block to Uncertain Model

This example shows how to make a Simulink® block linearize to an uncertain variable at the command line. To learn how to specify an uncertain block linearization using the Simulink model editor, see “Specify Uncertain Linearization for Core or Custom Simulink Blocks”.

For this example, open the Simulink model `slexAircraftExample`.

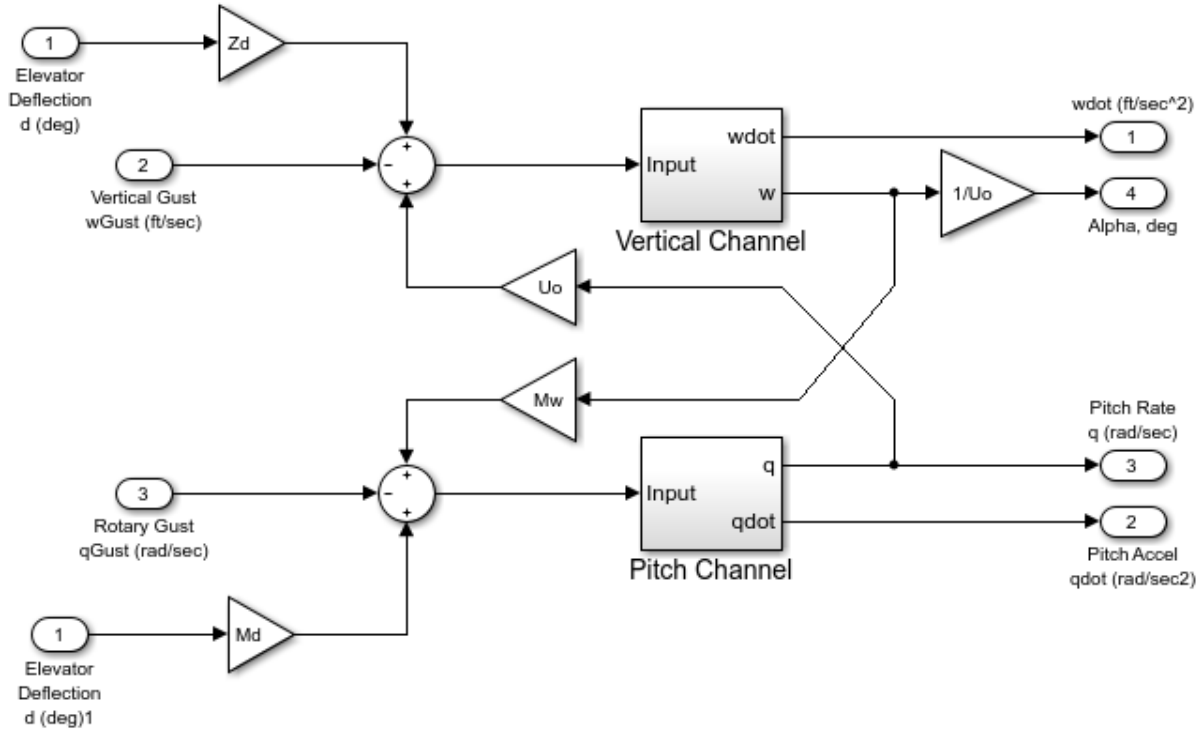
```
mdl = 'slexAircraftExample';
open_system(mdl)
```





Examine the subsystem `Aircraft Dynamics Model`.

```
subsys = [mdl,'/Aircraft Dynamics Model'];
open_system(subsys)
```



Suppose you want to specify the following uncertain real values for the gain blocks `Mw` and `Zd`.

```
Mw_unc = ureal('Mw',-0.00592,'Percentage',50);
Zd_unc = ureal('Zd',-63.9979,'Percentage',30);
```

To specify these values as the linearization for these blocks, create a `BlockSubs` structure to pass to the `linearize` function. The field names are the names of the Simulink blocks, and the values are the corresponding uncertain values. Note that in this model, the name of the `Mw` block is `Gain4`, and the name of the `Zd` block is `Gain5`.

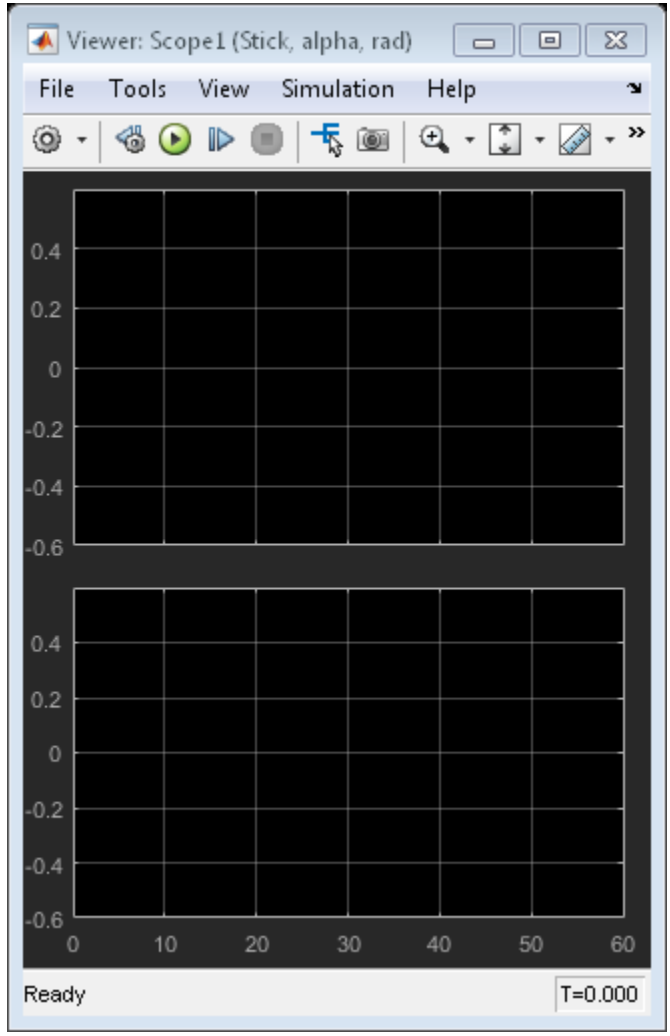
```
Mw_name = [subsys,'/Gain4'];
Zd_name = [subsys,'/Gain5'];

BlockSubs(1).Name = Mw_name;
BlockSubs(1).Value = Mw_unc;
```

```
BlockSubs(2).Name = Zd_name;
BlockSubs(2).Value = Zd_unc;
```

Compute the uncertain linearization. `linearize` linearizes the model at operating point specified in the model, making the substitutions specified by `BlockSubs`. The result is an uncertain state-space model with an uncertain real parameter for each of the two uncertain gains.

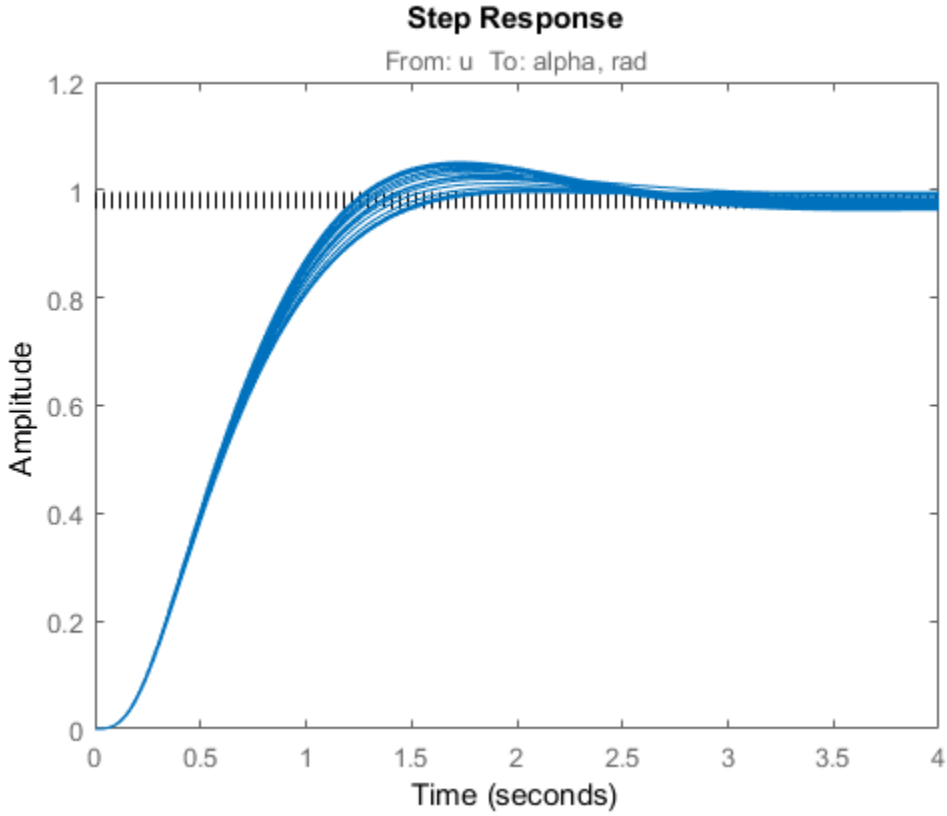
```
sys = linearize(mdl,BlockSubs)
```

```
sys =

  Uncertain continuous-time state-space model with 1 outputs, 1 inputs, 7 states.
  The model uncertainty consists of the following blocks:
    Mw: Uncertain real, nominal = -0.00592, variability = [-50,50]%, 1 occurrences
    Zd: Uncertain real, nominal = -64, variability = [-30,30]%, 1 occurrences

Type "sys.NominalValue" to see the nominal value, "get(sys)" to see all properties, and
```



Examine the undertain model response.

```
step(sys)
```



`step` takes random samples and provides a sense of the range of responses within the uncertainty of the linearized model.

See Also

`linearize`

Related Examples

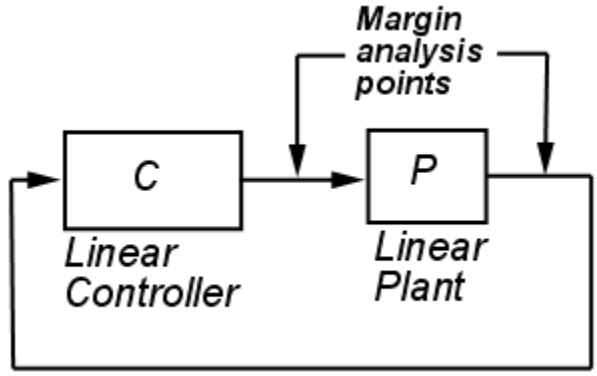
- "Specify Uncertain Linearization for Core or Custom Simulink Blocks" on page 5-21
- "Obtain Uncertain State-Space Model from Simulink Model" on page 5-20

Analyzing Stability Margin of Simulink Models

Robust Control Toolbox provides the `loopmargin` command to analyze the stability margins of LTI models created in MATLAB and Simulink models. To use `loopmargin` with Simulink models, you must have the Simulink Control Design software. This section describes the difference between the MATLAB and Simulink approaches of using `loopmargin` and the workflow for computing the stability margin of Simulinkmodels. For more information on how to analyze the stability margins of LTI models, see "MIMO Robustness Analysis".

How Stability Margin Analysis Using Loopmargin Differs Between Simulink and LTI Models

When analyzing stability margins of LTI models using the syntax `[cm,dm,mm] = loopmargin(P,C)`, the software assumes the input and output of the linear plant P as the margin analysis points, as shown in the following figure.



Analyzing stability margin of Simulink models differs from analyzing stability margin of LTI models because you can enter specific margin analysis points in the Simulink model. For more information on how to assign margin analysis points in Simulink models, see the "Usage with Simulink" section of the `loopmargin` reference page.

Stability Margin of Simulink Model

The `loopmargin` command computes the following types of stability margins:

- Loop-at-a-time classical gain and phase margins
- Loop-at-a-time disk margins
- Multi-loop disk margin

To learn more about these stability margins, see the "Algorithms" section of the `loopmargin` reference page.

The `loopmargin` command computes the stability margin based on linearization of Simulink models. To compute stability margins of a Simulink model:

1. Specify the block where you want to define a margin analysis point.
2. Specify the output port of the block where you want the margin analysis point.

   The software performs the analysis by opening the loop at all specified margin analysis point.
3. Use the `loopmargin` command to compute the stability margins at the margin analysis point.

Optionally, you can compare the classical gain and phase margins obtained using `loopmargin` with the stability margins computed for the linearized model. The results using the two approaches should match for simple SISO models. For MIMO models, the `loopmargin` command provides richer robustness information. For an example, see "Stability Margin of a Simulink Model" on page 5-33.

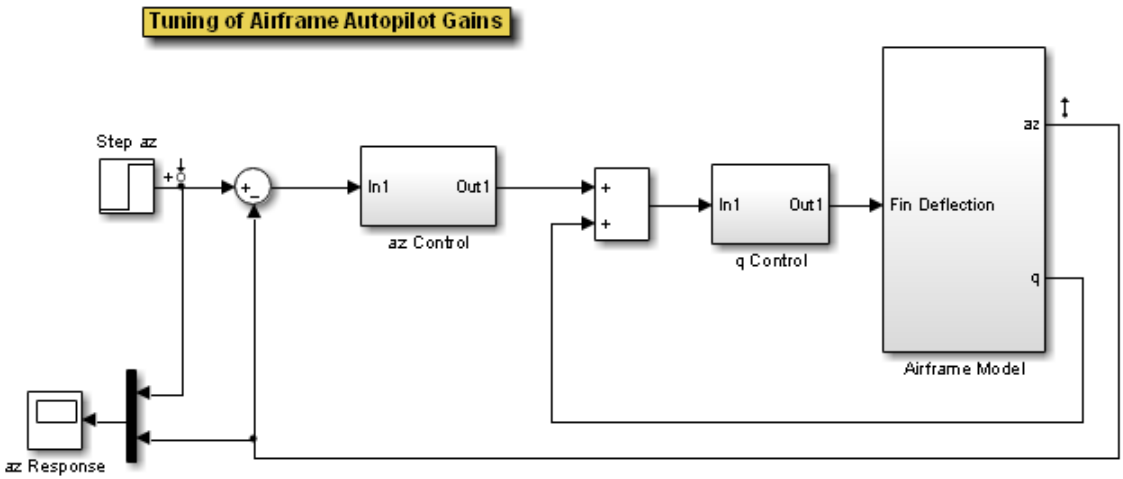
Additionally, you can compute stability margins by specifying multiple margin analysis points and multiple operating points. For an example, see "Loop Margins for an Airframe Autopilot".

Stability Margin of a Simulink Model

This example illustrates how to compute the stability margins of the `airframemargin` model and compare the results with stability margins computed using the linearized model.

1 Open the Simulink model:

```
airframemargin
```



2 Define a margin analysis point at the output of the Airframe Model block.

```
block1 = 'airframemargin/Airframe Model';
```

3 Specify the output `az` of the Airframe Model block as a margin analysis point.

```
port1 = 1;
```

4 Compute stability margins.

```
[cm,dm,mm] = loopmargin('airframemargin',block1,port1);
```

5 View the classical gain and phase margins.

```
cm
```

```
cm =
```

```
     GainMargin: [4.5652 2.5055e+003]
    GMFrequency: [7.1979 314.1593]
    PhaseMargin: 65.1907
    PMFrequency: 2.1463
    DelayMargin: 53.0113
    DMFrequency: 2.1463
         Stable: 1
```

6 Compare the classical gain and phase margins `cm` with stability margins of the linearized model computed using `allmargin`:

```
% Define linearization I/O points.
io = linio('airframemargin/Airframe Model',1,'looptransfer');
% Linearize the model.
lin_sys = linearize('airframemargin',io);
% Compute gain and phase margins.
cm_lin = allmargin(-lin_sys)
```

```
cm_lin =

     GainMargin: [4.5652 2.5055e+003]
    GMFrequency: [7.1979 314.1593]
    PhaseMargin: 65.1907
    PMFrequency: 2.1463
    DelayMargin: 53.0113
    DMFrequency: 2.1463
         Stable: 1
```

The gain and phase margins, `cm` and `cm_lin`, computed using the two approaches match.